Fundamentals of Communications Engineering

Department of Communications Engineering, College of Engineering, University of Diyala, 2016-2017

Class: Second Year
Instructor: Dr. Montadar Abas Taher
Room: Comm-02

Lecture: 08

Fourier Transform Theorems

* Fourier transform without its theorems (properties), is useless, just like a car without fuel.
* Lets call these theorems as properties throughout this class.

Property 1 : Linearity or Superposition

for any constant real/complex $a_1, a_2, \ldots$

$$\boxed{a_1 g_1(t) + a_2 g_2(t) \xleftrightarrow{F.T.} a_1 G_1(f) + a_2 G_2(f)}$$

Property 2 : Complex Conjugate

* If $g(t)$ is complex-valued signal, then

$$g(t) \xleftrightarrow{F.T.} G(f)$$

$$g^*(t) \xleftrightarrow{FT} G^*(-f)$$

* If $g(t)$ is a real-valued signal, then

$$g(t) = g^*(t) \xleftrightarrow{FT} G(f) = G^*(-f)$$

Property 3 : Duality theorem

if $g(t) \xleftrightarrow{FT} G(f)$

then $G(t) \longleftrightarrow g(-f)$

This relation can helps, very much, to solve or to find the time-domain function from the frequency-domain, which is sometimes very difficult integration, and vice versa.

Property 4 : Time Scaling

if $g(t) \xleftrightarrow{FT} G(f)$

then $g(\alpha t) \xleftrightarrow{FT} \frac{1}{|\alpha|} G\left(\frac{f}{\alpha}\right)$

Thus : Time compression $\longleftrightarrow$ Frequency Expanssion

Time expanssion $\longleftrightarrow$ Frequency compression

Property 5: Time Shifting (Delay)

* if the signal is time-shifted in time-domain, then the corresponding magnitude spectrum did not affected, but there will be a phase shift in the frequency domain.

Hence $g(t) \xleftrightarrow{F.T.} G(f)$

$$g(t \mp a) \longleftrightarrow G(f)\, e^{\mp j2\pi f a}$$

Property 6: Frequency Shifting

* Frequency shifting OR Modulation Theorem

$$g(t) \longleftrightarrow G(f)$$

$$g(t)\, e^{\mp j2\pi f_c t} \longleftrightarrow G(f \pm f_c)$$

Property 7 : Differentiation

$$g(t) \xleftrightarrow{F.T.} G(f)$$

$$\boxed{\frac{d^n}{dt^n} g(t) \xleftrightarrow{FT} (j2\pi f)^n G(f)}$$

NOTE : The physical meaning of differentiation

* Differentiation enhances the high frequency components of a signal.
* In other words, differentiation accentuates time variations.

Property 8 : Integration

$$g(t) \xleftrightarrow{FT} G(f)$$

$$\int_{-\infty}^{t} g(\tau)\, d\tau \xleftrightarrow{FT} \frac{1}{j2\pi f} G(f)$$

* Integration smoothes out the high-frequency components. In other words, suppression of the high-frequency components of the Signal.

Property 9: Convolution

$$g(t) \xleftrightarrow{FT} G(f)$$

$$h(t) \xleftrightarrow{FT} H(f)$$

then $$g(t) \circledast h(t) \xleftrightarrow{FT} G(f)\,H(f)$$

and

$$g(t)\,h(t) \xleftrightarrow{FT} G(f) \circledast H(f)$$

Thus: In general

Convolution $\xleftrightarrow{FT}$ Multiplication

Multiplication $\xleftrightarrow{FT}$ Convolution

Property 10 : Area under g(t).

area under curve stands for average value or the D.C. value.

$$\int_{-\infty}^{\infty} g(t)\,dt = G(0)$$

Property 11 : Area under G(f)

$$g(0) = \int_{-\infty}^{\infty} G(f)\,df$$

Property 12 : Rayleigh's Energy Theorem

$$\int_{-\infty}^{\infty} |g(t)|^2\,dt = \int_{-\infty}^{\infty} |G(f)|^2\,df$$