Fundamentals of Communications Engineering

Department of Communications Engineering, College of Engineering, University of Diyala, 2016-2017

Class: Second Year
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Room: Comm-02

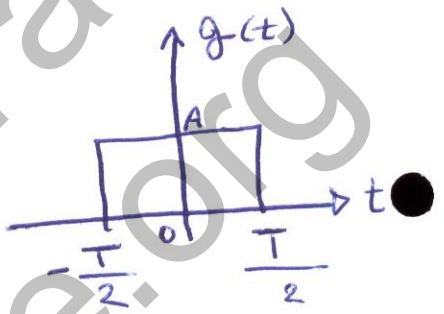
Lecture: 07

Fourier Transformation Pairs

$$g(t) \xleftrightarrow{F.T.} G(f)$$

① Rectangular function, Box function, Boxer function, Gate function.

$$g(t) = A\ rect\left(\frac{t}{T}\right) \longrightarrow$$



$$\boxed{G(f) = AT\ sinc(fT)}$$

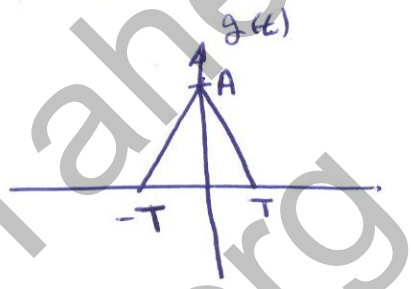
$$A\ rect\left(\frac{t}{T}\right) = A\ rect_T(t) = A\Pi\left(\frac{t}{T}\right)$$

$$AT\ sinc(fT)$$

② Triangular Function

$$tri\left(\frac{t}{T}\right) = \Lambda\left(\frac{t}{T}\right) \overset{F.T.}{\longleftrightarrow} sinc^2(f)$$

$$tri\left(\frac{t}{T}\right) = \begin{cases} 1-|t| & |t| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

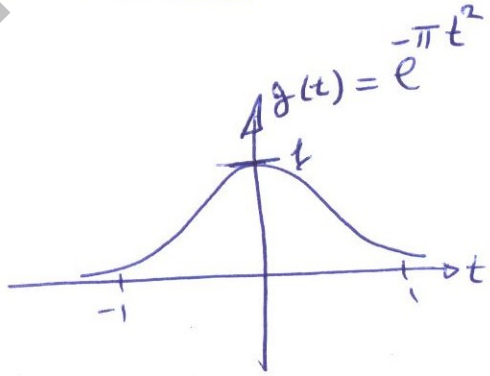


Generally

$$A\,tri\left(\frac{t}{T}\right) \overset{F.T.}{\longleftrightarrow} AT\,sinc^2(fT)$$

③ Gaussian Function

$$e^{-\pi t^2} \overset{F.T.}{\longleftrightarrow} e^{-\pi f^2}$$



④ The Constant Function:

$g(t) = A$

$$A \xleftrightarrow{F.T.} A\delta(f)$$
$$\text{or} \quad 2\pi A\delta(\omega)$$

⑤ Impulse Function

Delta Function $\delta(t-c)$

$$\delta(t \pm a) \xleftrightarrow{F.T.} e^{\pm j2\pi fa}$$

⑥ Complex Exponential: $g(t) = e^{\pm j2\pi f_0 t}$

$$e^{\pm j2\pi f_0 t} \xleftrightarrow{F.T.} \delta(f \mp f_0)$$

OR

$$e^{\pm j\omega_0 t} \xleftrightarrow{F.T.} 2\pi\delta(\omega \mp \omega_0)$$

(7) Cosine Function

$$g(t) = \cos(2\pi f_0 t) = \frac{1}{2}\left[e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}\right]$$

$$g(t) = \cos(2\pi A t) = \frac{1}{2}\left[e^{j2\pi A t} + e^{-j2\pi A t}\right]$$

$$\cos(2\pi f_c t) \overset{F.T.}{\longleftrightarrow} \frac{1}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$$

OR

$$\cos(\omega_c t) \overset{F.T.}{\longleftrightarrow} \pi\left[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right]$$

(8) Sine Function

$$\sin(2\pi f_c t) \overset{F.T.}{\longleftrightarrow} \frac{1}{2j}\left[\delta(f-f_c) - \delta(f+f_c)\right]$$

OR

$$\sin(\omega_c t) \overset{F.T.}{\longleftrightarrow} \frac{\pi}{j}\left[\delta(\omega-\omega_c) - \delta(\omega+\omega_c)\right]$$

(9) Unit Step Function

$$u(t) \xleftrightarrow{F.T.} \frac{1}{j2\pi f} + \frac{\delta(f)}{2}$$

OR

$$u(t) \xleftrightarrow{F.T.} \frac{1}{j\omega} + \pi\delta(\omega)$$

(10) Signum Function

$$sgn(t) = \frac{|t|}{t} = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$

$$sgn(t) \xleftrightarrow{F.T.} \frac{1}{j\pi f}$$

OR

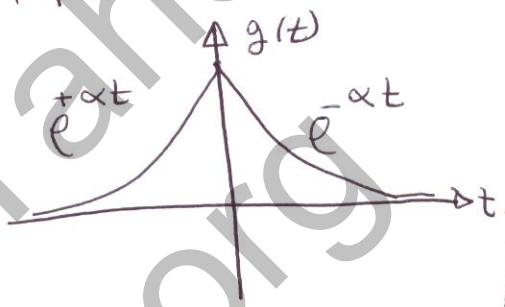
$$sgn(t) \xleftrightarrow{F.T.} \frac{2}{j\omega}$$

(11) Decay Exponential Function

* Note : the decay exponential function is not a sinusoidal.

$$g(t) = e^{\pm \alpha t}$$

there is no j in the exponent.
it is a real function.



$$e^{\pm \alpha t} \overset{FT.}{\longleftrightarrow} \frac{1}{\alpha \mp j2\pi f}$$

OR

$$e^{\pm \alpha t} \overset{FT.}{\longleftrightarrow} \frac{1}{\alpha \mp j\omega}$$

Furthermore :

$$e^{\pm a|t|} \overset{FT}{\longleftrightarrow} \frac{2a}{a^2 \mp \omega^2}$$