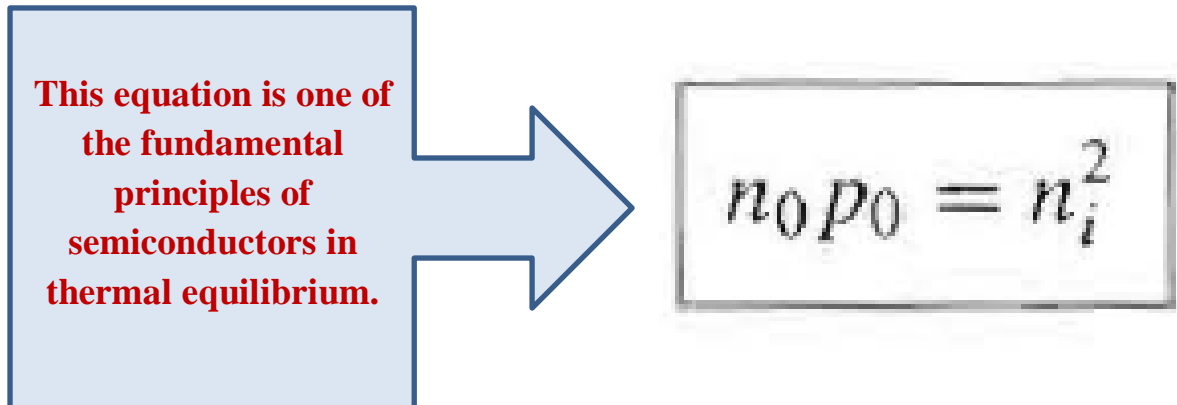


Lecture (8)The mass action law

In a semiconductor the law of mass action states that the product of the electron concentration  $n_0$  and the hole concentration  $p_0$  is always equal to the square of the intrinsic carrier concentration (at a given temperature), i.e.:



This law is valid for extrinsic semiconductors (with impurities) as well as for intrinsic semiconductors

- If  $N_d$  is the concentration of donor atoms, it is a positive charges per cubic meter are contributed by the donor ion ,hence the total positive charges density equal  $(N_d + p_0)$ .similarly when  $N_a$  is the concentration of acceptor ions these contributed  $(N_a + n_0)$  is negative charges per cubic meter ,the total negative charge density is  $(N_a + n_0)$
- In thermal equilibrium, the semiconductor crystal is electrically neutral • The charge neutrality condition is expressed by equating

$$n_0 + N_a = p_0 + N_d$$

The intrinsic carrier concentration:  $n_i = p_i$  (same density of free electrons and holes in an intrinsic semiconductor).

The extrinsic n-type semiconductor: 
$$\begin{cases} n_n = N_D \\ p_n = \frac{n_i^2}{N_D} \end{cases}$$

$$p = \frac{n_i^2}{N_d} \ll N_d$$

The extrinsic p-type semiconductor 
$$\begin{cases} n_p = \frac{n_i^2}{N_A} \\ p_p = N_A \end{cases}$$

$$n = \frac{n_i^2}{N_a} \ll N_a$$

With  $n$ ,  $p$  respectively the electron and hole density ;  $N_{D,A}$  respectively the donor and acceptor doping density (concentration).

p-type material at equilibrium

$$p_v \approx N_A$$

$$n_c \approx \frac{n_i^2}{N_A}$$

n-type material at equilibrium

$$n_c \approx N_D$$

$$p_v \approx \frac{n_i^2}{N_D}$$

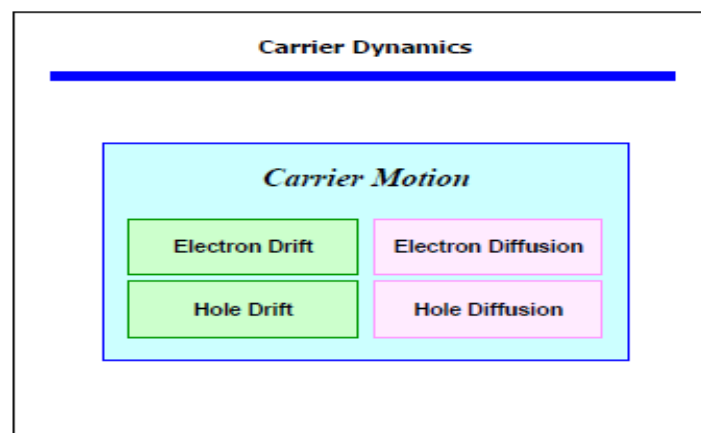
Example

- Calculate the carrier concentration in silicon doped with  $10^{16}$  atoms/cm<sup>3</sup> of As at 300°K

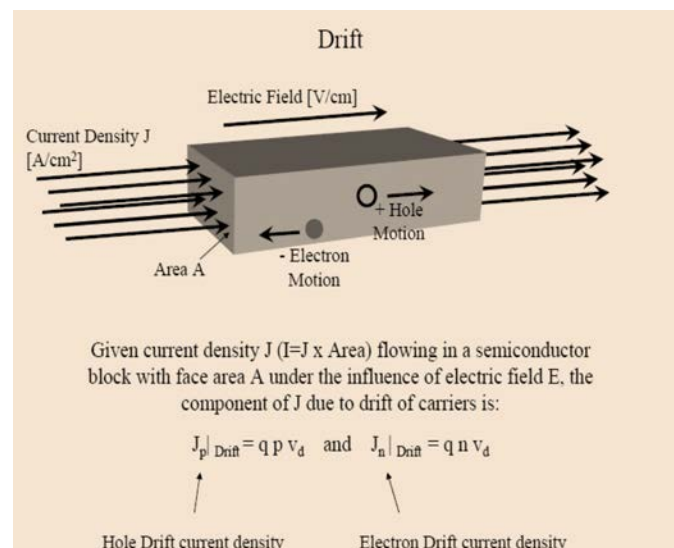
- $n_i = 9.65 \times 10^9$
- $n_0 = N_D = 10^{16} \text{ cm}^{-3}$
- $p_0 = n_i^2 / N_D = (9.65 \times 10^9)^2 / 10^{16} \text{ cm}^{-3} = 9.31 \times 10^3$

Conduction System in Semiconductor:

We have two types of carriers one carrier is negative (the free electron) with mobility ( $\mu_n$ ) and the other is positive (the holes) with mobility ( $\mu_p$ ), these particles move in opposite. There are two current mechanisms which cause charges to move in semiconductors. The two mechanisms we shall study in this chapter are drift and diffusion, and both electrons and holes conduct current in semiconductor

1-Drift Motion (carriers drift by electric field):

To understand drift, assume an electric field is applied to a semiconductor. The field produces a force that acts on free electrons and holes, which then experience a net drift velocity and net movement.



Consider an n-type semiconductor with a large number of free electrons. An electric field  $E$  applied in one direction produces a force on the electrons in the *opposite* direction, because of the electrons' negative charge.

The electrons acquire a drift velocity  $v_{dn}$  (in cm/s) which can be written as

$$v_{dn} = -\mu_n E$$

where  $\mu_n$  is a constant called the **electron mobility** and has units of  $\text{cm}^2/\text{V.s}$ . For low-doped silicon, the value of  $\mu_n$  is typically  $1350 \text{ cm}^2/\text{V.s}$ . The mobility can be thought of as a parameter indicating how well an electron can move in a semiconductor. The negative sign in above Equation indicates that the electron drift velocity is opposite to that of the applied electric field as shown in above Figure. The electron drift produces a drift current density  $J_n$  ( $\text{A}/\text{cm}^2$ ) given by

$$J_n = -en v_{dn} = -en(-\mu_n E) = +en\mu_n E$$

where  $n$  is the electron concentration ( $\#/\text{cm}^3$ ) and  $e$ , in this context, is the magnitude of the electronic charge. The conventional drift current is in the opposite direction from the flow of negative charge, which means that the drift current in an n-type semiconductor is in the same direction as the applied electric field.

Next consider a p-type semiconductor with a large number of holes. An electric field  $E$  applied in one direction produces a force on the holes in the *same* direction, because of the positive charge on the holes. The holes acquire a drift velocity  $v_{dp}$  (in cm/s), which can be written as

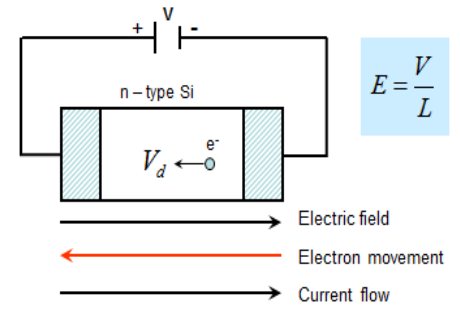
$$v_{dp} = +\mu_p E$$

where  $\mu_p$  is a constant called the **hole mobility**, and again has units of  $\text{cm}^2/\text{V.s}$

The positive sign in above Equation indicates that the hole drift velocity is in the same direction as the applied electric field as shown in above Figure.

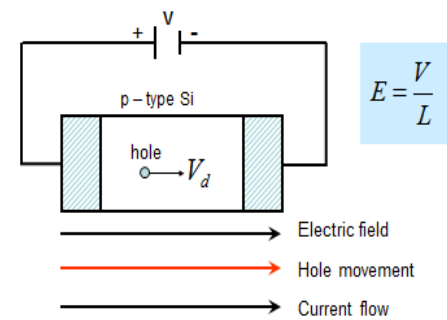
The hole drift produces a drift current density  $J_p$  ( $\text{A}/\text{cm}^2$ ) given by where  $p$  is the hole concentration ( $\#/\text{cm}^3$ ) and  $e$  is again the magnitude of the electronic charge. The conventional drift current is in the same direction as the flow of positive charge, which means that the drift current in a p-type material is also in the same direction as the applied electric field. The hole drift produces a drift current density  $J_p$  ( $\text{A}/\text{cm}^2$ ) given by

#### ❖ n - type Si



Current carriers are mostly electrons.

#### ❖ p - type Si



Current carriers are mostly holes.

$$J_p = +epv_{dp} = +ep(+\mu_p E) = +ep\mu_p E$$

Since a semiconductor contains both electrons and holes, the total drift current density is the sum of the electron and hole components. The total drift current density is then written as

$$J = en\mu_n E + ep\mu_p E = \sigma E = \frac{1}{\rho} E$$

The resistivity for two types of materials

- In n-type semiconductor:

$$\rho_n \simeq \frac{1}{qN_d\mu_n}$$

- In p-type semiconductor:

$$\rho_p \simeq \frac{1}{qN_a\mu_p}$$

The total conductivity of a semiconductor is:

$$\sigma = en\mu_n + ep\mu_p$$

### ❖ Drift Current Equations

For undoped or intrinsic semiconductor ;  $n=p=n_i$

For electron

$$J_n = nqE\mu_n$$

drift current for electrons

number of free electrons per unit volume

mobility of electron

For hole

$$J_p = pqE\mu_p$$

drift current for holes

number of free holes per unit volume

mobility of holes

### ❖ Drift Current Equations

Total current density

$$J_i = J_n + J_p$$

$$J_i = nqE\mu_n + pqE\mu_p$$

since

$$n = p = n_i$$

$$J_i = n_i q (\mu_n + \mu_p) E$$

For a pure intrinsic semiconductor

For Extrinsic material

n-type semiconductor;

$$n \gg p \Rightarrow J_T \cong nq\mu_n E = N_D q\mu_n E$$

where  $N_D$  is the shallow donor concentration

p-type semiconductor;

$$p \gg n \Rightarrow J_T \cong pq\mu_p E = N_A q\mu_p E$$

where  $N_A$  is the shallow acceptor concentration

## 2-Diffusion Motion(Carriers diffuse to the area at lower carrier concentration):

**Diffusion** : Particles tend to spread out or redistribute from areas of high concentration to areas of lower concentration.

→ It is possible for an electric current to flow in a semiconductor even in the absence of the applied voltage provided a concentration gradient exists in the material.

→ A concentration gradient exists if the number of either elements or holes is greater in one region of a semiconductor as compared to the rest of the Region.

→ In a semiconductor material the charge carriers have the tendency to move from the region of higher concentration to that of lower concentration of the same type of charge carriers. Thus the movement of charge carriers takes place resulting in a current called diffusion current.

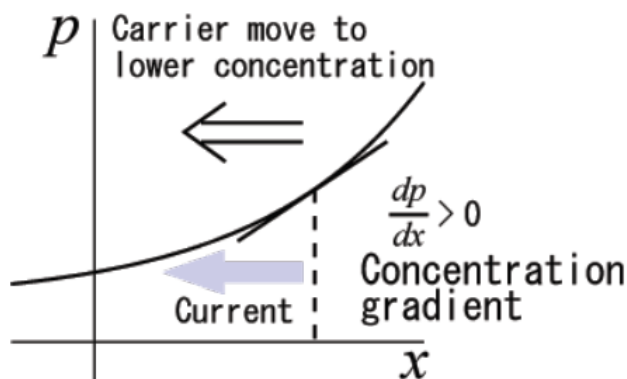
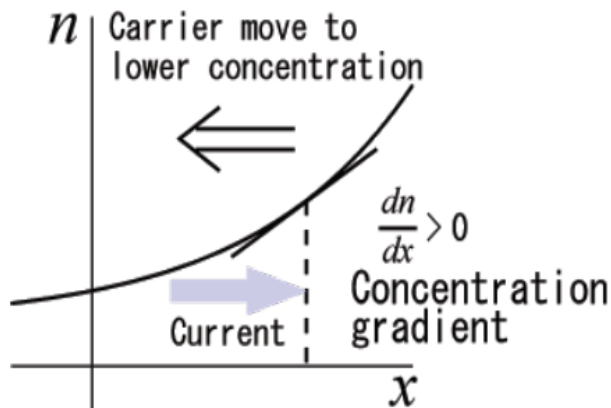
In the diffusion process, Statistically, we can assume that, at any particular instant, approximately half of the particles in the high-concentration region are moving *away* from that region toward the lower-concentration region. We can also assume that, at the same time, approximately half of the particles in the lower concentration region are moving *toward* the high-concentration region. However, by definition, there are fewer particles in the lower-concentration region than there are in the high-concentration region. Therefore, the net result is a flow of particles away from the high-concentration region and toward the lower concentration region. This is the basic diffusion process. The diffusion current density due to the diffusion of electrons can be written as

$$J_n = eD_n \frac{dn}{dx}$$

where  $e$ , in this context, is the magnitude of the electronic charge,  $dn/dx$  is the gradient of the electron concentration, and  $D_n$  is the **electron diffusion coefficient**. The diffusion current density due to the diffusion of holes can be written as (for one dimension)

$$J_p = -eD_p \frac{dp}{dx}$$

where  $e$  is still the magnitude of the electronic charge,  $dp/dx$  is the gradient of the hole concentration, and  $D_p$  is the **hole diffusion coefficient**. Note the change in sign between the two diffusion current equations. This change in sign is due to the difference in sign of the electronic charge between the negatively charged electron and the positively charged hole.



## Fick's first law-

### Key diffusion relationship

$$\text{Diffusion flux} \propto -\text{concentration gradient}$$

**Flux**  $\equiv$  number of particles crossing a unit area per unit time [ $\text{cm}^{-2} \cdot \text{s}^{-1}$ ]

For Electrons:

$$F_n = -D_n \frac{dn}{dx}$$

For Holes:

$$F_p = -D_p \frac{dp}{dx}$$

$D_n \equiv$  electron diffusion coefficient [ $\text{cm}^2 \text{s}^{-1}$ ]

$D_p \equiv$  hole diffusion coefficient [ $\text{cm}^2 \text{s}^{-1}$ ]

$D$  measures the ease of carrier diffusion in response to a concentration gradient:  $D \uparrow \Rightarrow F^{\text{diff}} \uparrow$

$D$  limited by vibration of lattice atoms and ionized dopants.

## Total Current Density

In general, total current can flow by drift and diffusion separately. **Total current density:**

$$J_n = J_n^{\text{drift}} + J_n^{\text{diff}} = qn\mu_n E + qD_n \frac{dn}{dx}$$

$$J_p = J_p^{\text{drift}} + J_p^{\text{diff}} = qp\mu_p E - qD_p \frac{dp}{dx}$$

$$J_{\text{total}} = J_n + J_p$$

## ❖ Einstein Relation

Einstein relation relates the two independent current mechanisms of mobility with diffusion;

$$\frac{D_n}{\mu_n} = \frac{kT}{q} \quad \text{and} \quad \frac{D_p}{\mu_p} = \frac{kT}{q} \quad \text{for electrons and holes}$$

Constant value at a fixed temperature

$$\frac{\text{cm}^2/\text{sec}}{\text{cm}^2/\text{V} - \text{sec}} = \text{volt} \quad \frac{kT}{q} = \frac{(J/K)(K)}{C} = \text{volt}$$

$$\frac{kT}{q} = 25 \text{ mV} \quad \text{at room temperature}$$

**Carriers life time :**

In pure S.c (n=p) ,thermal agitation continues to produce new hole-electrons pairs while other e-h pairs disappear as a result of recombination on an average ,a hole or electron will exist for  $(\tau_p)$   $(\tau_n)$  sec before recombination.

$(\tau_p)$   $(\tau_n)$ : mean lifetime of holes and electrons carrier life time range from  $(10^{-9} \text{ sec})$  to hundreds of  $(10^{-6} \text{ sec})$  and it is indicated the time required for (n) and (p) concentration which have been caused to change to return their equilibrium concentration

The equilibrium concentration of holes and electrons in the n-type bar ( $p^0$  &  $n^0$ ) , and the new concentration are ( $p^-$  &  $n^-$ ) under the influence of the radiation.

The excess concentration is  $(p^- - p^0)$  for holes and  $(n^- - n^0)$  for electrons

$$p^- - p^0 = n^- - n^0$$

**Diffusion length,  $L$** 

When excess carriers are generated in a specimen, the minority carriers diffuse a distance, a characteristic length, over which minority carriers can diffuse before recombining majority carriers. This is called as a diffusion length,  $L$ .



Diffusion length for electrons

$$L_n = \sqrt{D_n \tau_n}$$

Diffusion length for holes

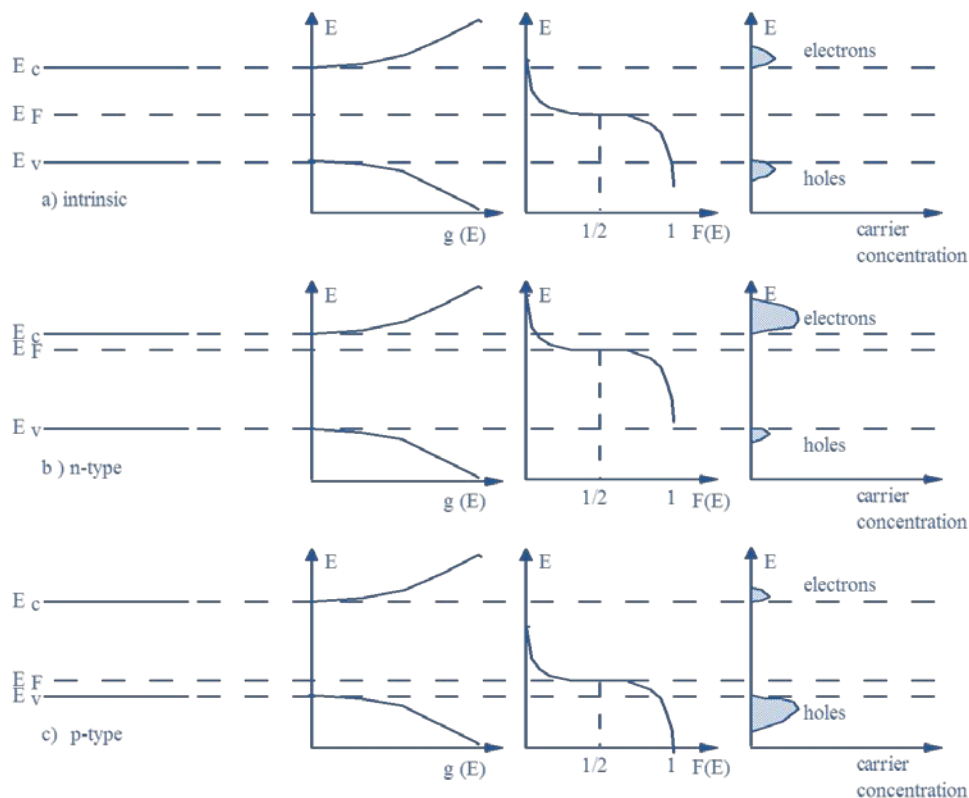
$$L_p = \sqrt{D_p \tau_p}$$

## Energy band diagrams

As a consequence of the mathematical description of the wave character of carriers in semiconductors, Free electrons have energies in an allowed energy band higher in energy than the bandgap, the conduction band  $E_c$ , whilst free holes have energies in an allowed energy band lower in energy than the bandgap, the valence band  $E_v$ .

The Fermi-Dirac distribution function  $f(E)$  as illustrated in below Figure  $g(E)$  gives the distribution of energy levels (states) as a function of energy.  $f(E)$  gives the probability of finding an electron at energy  $E$ . Thus  $1-f(E)$  gives the probability of finding a hole or electrons and it is changing according the types of material.

- 1- for Intrinsic material the concentration of electron in the conduction band equal the concentration of holes in valance band , therefore the Fermi-energy ( $E_F$ ) must lie at the middle of the band gab .
- 2- In n-type there is high concentration of electrons in the conduction band compared with the hole concentration in the valance band .thus in n-type material the distribution function  $f(E)$  must lie above its intrinsic position on the energy scale .
- 3- In p-type material the hole concentration in the valance band is higher tha the concentration of electrons in conduction band , thus the Fermi level lies near the valance band .



**Figure: The energy band diagram with the position of the Fermi level  $E_F$  as a function of the doping type (left). The integration of the product of the density of states  $g(E)$  and the Fermi-Dirac distribution function  $f(E)$  gives the carrier distribution over the energy range above or below the bandgap (right).**

**Example:** A Si sample is doped with  $10^{-4}$  atomic% of P donors. Assuming complete ionisation of donors at room temperature, calculate the charge carrier concentration and conductivity at room temperature.

[For Si:  $\rho = 2330 \text{ kg m}^{-3}$ , atomic weight = 28,  $\mu_e = 0.15 \text{ m}^2\text{V}^{-1}\text{s}^{-1}$ ]

**Answer**

$$\frac{N_d}{N_{\text{Si}}} = 10^{-6}$$

where  $N_{\text{Si}}$  – number of Si atoms per unit volume Obtain  $N_{\text{Si}}$  from Avagadro's No.

$$N_{\text{Si}} = \frac{\rho}{28 \times 10^{-3}} \times 6 \times 10^{23}$$

This gives  $N_{\text{Si}} = 5 \times 10^{28} \text{ m}^{-3}$

$$N_d = 5 \times 10^{22} \text{ m}^{-3}.$$

Complete ionisation, n-type semiconductor: So charge carrier concentration is

$$n_e = N_d = 5 \times 10^{22} \text{ m}^{-3}$$

(neglect  $p_v$ )

Conductivity  $\sigma$   $\sigma = n_e e \mu_e$

[hole contribution negligible]

So  $\sigma = 5 \times 10^{22} \times 1.6 \times 10^{-19} \times 0.15 \text{ cm}^{-1} \text{ m}^{-1}$

$$= 1200 \Omega^{-1} \text{ m}^{-1}$$

#### Example 1.4

Calculate the diffusion current density for a given semiconductor.

Consider silicon at  $T = 300 \text{ K}$ . Assume the electron concentration varies linearly from  $n = 10^{12} \text{ cm}^{-3}$  to  $n = 10^{16} \text{ cm}^{-3}$  over the distance from  $x = 0$  to  $x = 3 \mu\text{m}$ . Assume  $D_n = 35 \text{ cm}^2/\text{s}$ .

We have

$$J_n = e D_n \frac{dn}{dx} = e D_n \frac{\Delta n}{\Delta x} = (1.6 \times 10^{-19})(35) \left( \frac{10^{12} - 10^{16}}{0 - 3 \times 10^{-4}} \right)$$

or

$$J_n = 187 \text{ A/cm}^2$$

*Comment:* Diffusion current densities on the order of a few hundred amperes per square centimeter can also be generated in a semiconductor.

Example: Dopant Compensation

Consider the same Si sample, doped *additionally* with  $10^{17}/\text{cm}^3$  Arsenic. What is its resistivity?

Answer:

$$N_A = 10^{16}/\text{cm}^3, N_D = 10^{17}/\text{cm}^3 \quad (N_D \gg N_A \rightarrow \text{n-type})$$

$$\rightarrow n \approx 9 \times 10^{16}/\text{cm}^3 \quad \text{and} \quad p \approx 1.1 \times 10^3/\text{cm}^3$$

$$\rho = \frac{1}{qn\mu_n + qp\mu_p} \cong \frac{1}{qn\mu_n}$$

$$= \left[ (1.6 \times 10^{-19})(9 \times 10^{16})(600) \right]^{-1} = 0.12 \, \Omega\text{-cm}$$

Example

Consider a Si sample doped with  $10^{16}/\text{cm}^3$  Boron. What is its resistivity?

Answer:

$$N_A = 10^{16}/\text{cm}^3, N_D = 0 \quad (N_A \gg N_D \rightarrow \text{p-type})$$

$$\rightarrow p \approx 10^{16}/\text{cm}^3 \quad \text{and} \quad n \approx 10^4/\text{cm}^3$$

$$\rho = \frac{1}{qn\mu_n + qp\mu_p} \cong \frac{1}{qp\mu_p}$$

$$= \left[ (1.6 \times 10^{-19})(10^{16})(450) \right]^{-1} = 1.4 \, \Omega\text{-cm}$$

H.W

Calculate the drift current density for a given semiconductor. Consider silicon at  $T = 300 \text{ K}$  doped with arsenic atoms at a concentration of  $N_d = 8 \times 10^{15} \text{ cm}^{-3}$ . Assume mobility values of  $\mu_n = 1350 \text{ cm}^2/\text{V.s}$  and  $\mu_p = 480 \text{ cm}^2/\text{V.s}$ . Assume the applied electric field is  $100 \text{ V/cm}$ .