

Lecture (6)Electronic and Ionic Conduction)

Electricity : it is a phenomenon associated with stationary or moving electric charges. Electric charge is a fundamental property of matter and is borne by elementary particles. In electricity the particle involved is the electron, which carries a charge designated, by convention, as negative. Thus, the various manifestations of electricity are the result of the accumulation or motion of numbers of electrons.

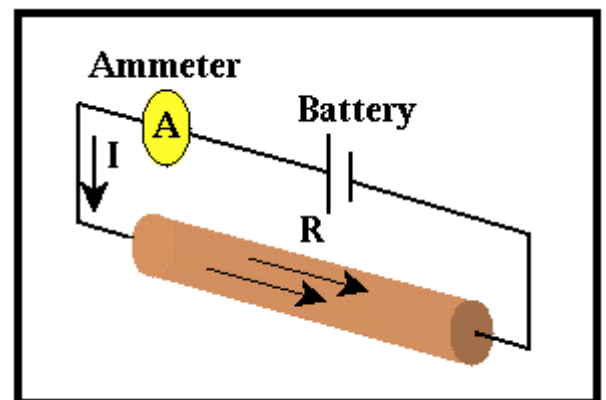
Electrical Conduction: Electrical conduction is the movement of electrically charged particles through a transmission medium. The movement can form an electric current in response to an electric field.

Ionic Conduction: -motion of anions and/or cations-solutions of electrolytes (salts, acids and bases) in water and other liquids, molten salts.

Ohm's Law

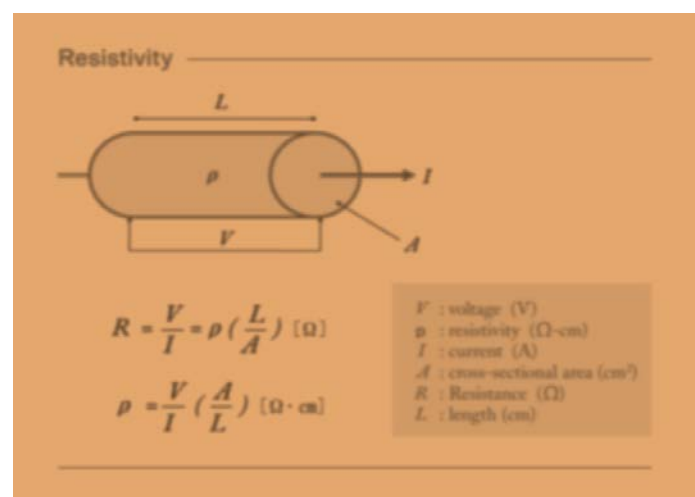
Ohm demonstrated that there are no perfect electrical conductors through a series of experiments in 1825. Every conductor he tested offered some level of resistance. These experiments led to Ohm's law. Ohm's law states the current equals voltage divided by resistance.

$$I = V/R$$



where "V" (the voltage or the electromotive force - measured in volts) equals "I" (the current - measured in amperes) times "R" (is property of any object or substance to resist or oppose the flow of an electrical current. The unit of resistance is the ohm).

Resistivity is defined as the resistance in the wire, times the cross-sectional area of the wire, divided by the length of the wire. The units associated with resistivity are thus, ohm - m (ohm - meters). The diagram bellow shows this equation as it would work with a common wire, represented by the tan cylinder.

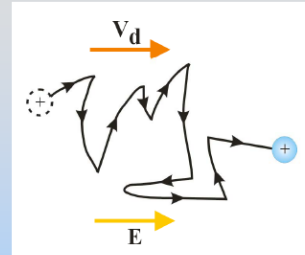


Conductivity vs Resistivity

Conductivity and resistivity are inversely proportional to each other. When conductivity is low, resistivity is high. When resistivity is low, conductivity is high. The equation is as follows:

$$\sigma = \frac{1}{\rho}$$

Conductivity and Resistivity



Ability of current to flow depends on density of charges & rate of scattering

Two quantities summarize this:

σ : conductivity

ρ : resistivity

Conductivity and Resistivity

- characterize material: $J = nev_d = ne \left(\frac{e\tau E}{m} \right) = \frac{ne^2\tau}{m} E$
conductivity, $\sigma = \frac{ne^2\tau}{m} \rightarrow J = \sigma E$
- current caused by E exerting forces on charge carriers, $\propto E, n, \tau$
- conductivity decreases with temperature (more collisions...)
- more practical: resistivity, $\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$
- Units: $\text{A/C/N m}^2 \equiv \Omega^{-1} \text{ m}^{-1}$ for σ ; $\Omega \text{ m}$ for ρ (Ω is ohm)

Example: A 2.0-mm-diameter aluminium wire carries a current of 800 mA
What is the electric field strength inside the wire?

$$E = \frac{J}{\sigma} = \frac{I}{\sigma A} = \frac{I}{\sigma \pi r^2} = \frac{0.80 \text{ A}}{(3.5 \times 10^{-7} \Omega^{-1} \text{ m}^{-1}) \pi (0.0010 \text{ m})^2}$$

$$= 0.00072 \text{ N/C}$$

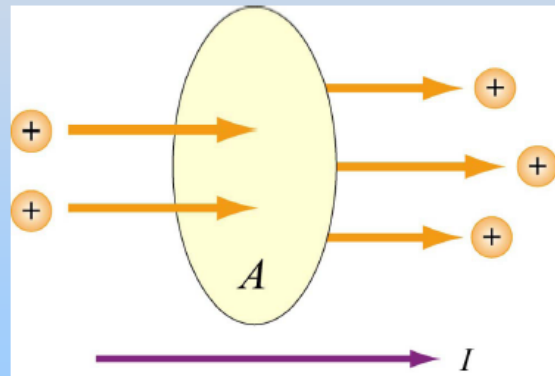
Current: Flow Of Charge

Average current I_{av} : Charge ΔQ flowing across area A in time Δt

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

Instantaneous current: differential limit of I_{av}

$$I = \frac{dQ}{dt}$$



Units of Current: Coulomb/second = Ampere

Mobility and Conductivity

In solid-state physics, the **electron mobility** characterizes how quickly an electron can move through a metal or semiconductor, when pulled by an electric field. In semiconductors, there is an analogous quantity for holes, called **hole mobility**. The term **carrier mobility** refers in general to both electron and hole mobility in semiconductors.

Electron and hole mobility are special cases of electrical mobility of charged particles in a fluid under an applied electric field.

When an electric field E is applied across a piece of material, the electrons respond by moving with an average velocity called the drift velocity, v_d . Then the electron mobility μ is defined by the equation.

Charge Mobility

- charge velocity in a conductor depends on the charge *mobility*

$$v_e = -\mu_e E, v_h = \mu_h E, v_i = \mu_i E, \text{ m/s}$$

- metals support electron current
drift electron velocity in metals: $v_d = -\mu_e E$
- semiconductors support both electron and hole currents
- most electrolytes support both electron and ion currents
- in general plasmas support both electron and ion currents
- mobility may in general depend on E (nonlinear conductors)

$$v_d = \mu E.$$

where:

E is the magnitude of the electric field applied to a material,

v_d is the magnitude of the electron drift velocity (in other words, the electron drift speed) caused by the electric field, and

μ is the electron mobility (is defined as the proportionality factor between the mean velocity of charged particles and the electric field strength).

usually, the electron drift velocity in a material is directly proportional to the electric field, which means that the electron mobility is a constant (independent of electric field). When this is not true (for example, in very large electric fields), the mobility depends on the electric field.

The SI unit of velocity is m/s, and the SI unit of electric field is V/m. Therefore the SI unit of mobility is (m/s)/(V/m) = m²/(V·s). However, mobility is much more commonly expressed in cm²/(V·s) = 10⁻⁴ m²/(V·s).

conductivity is

Current Density

If (N) electrons are contained in length (L) of conductor, and if takes an electrons atime (t) to travel a distance of (L) in the conductor .

The total numbers of electrons passing through any cross –section of a wire in unit time is (N/t), thus the total charge per second passing through it by definition, is the current in (amper) is

$$I = Nq/t = Nq v_d /L$$

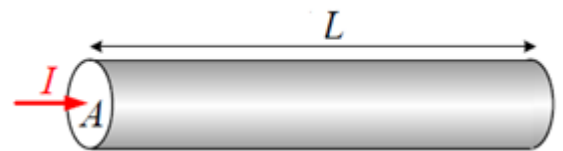
The current density (J) equal:

$$\mathbf{J} = \mathbf{I}/A$$

So

$$\mathbf{J} = Nq v_d / LA$$

Where (LA) is the volume containing the N-electrons, Also N / LA is the electron oncentration (n)

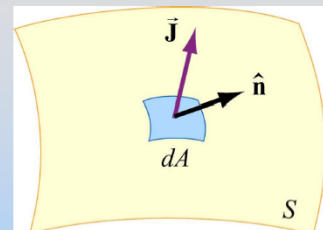


Current Density J

J: current/unit area

$$\vec{J} \equiv \frac{I}{A} \hat{\mathbf{I}}$$

$\hat{\mathbf{I}}$ points in direction of current



$$I = \int_S \vec{J} \cdot \hat{\mathbf{n}} dA = \int_S \vec{J} \cdot d\vec{A}$$

Relation of mobility with conductivity

Conductivity is proportional to the product of mobility and carrier concentration. For example, the same conductivity could come from a small number of electrons with high mobility for each, or a large number of electrons with a small mobility for each. For metals, it would not typically matter which of these is the case, since most metal electrical behavior depends on conductivity alone. Therefore mobility is relatively unimportant in metal physics. On the other hand, the Semiconductor mobility depends on the impurity concentrations (including donor and acceptor concentrations), defect concentration, temperature, and electron and hole concentrations. It also depends on the electric field, particularly at high fields when velocity saturation occurs.

There is a simple relation between mobility and electrical conductivity. Let n be the number density of electrons, and let μ_e be their mobility. In the electric field \mathbf{E} , each of these electrons will move with the velocity vector $-\mu_e \mathbf{E}$, for a total current density of $ne\mu_e \mathbf{E}$ (where e is the elementary charge). Therefore, the electrical conductivity σ satisfies:^[1]

$$\sigma = ne\mu_e.$$

This formula is valid when the conductivity is due entirely to electrons. In a p-type semiconductor, the conductivity is due to holes instead, but the formula is essentially the same: If p is the density of holes and μ_h is the hole mobility, then the conductivity is

$$\sigma = pe\mu_h.$$

If a semiconductor has both electrons and holes, the total

$$\sigma \equiv qp\mu_p + qn\mu_n$$

Thus the magnitude of conductivity depends on three factors:

- 1-The charge of carrier (q with unit C)
- 2-The mobility of charge carrier (μ with unit $\text{m}^2/\text{V}\cdot\text{s}$)
- 3- the number of charge carrier per unit volume (concentration n or p with unit cm^{-3})

Energy Distribution of electrons and Fermi-level

The Fermi energy is a concept in quantum mechanics usually referring to the energy difference between the highest and lowest occupied single-particle states in a quantum system of non-interacting fermions at absolute zero temperature. In a Fermi gas the lowest occupied state is taken to have zero kinetic energy, whereas in a metal the lowest occupied state is typically taken to mean the bottom of the conduction band. The Fermi-Dirac Distribution

The Fermi–Dirac distribution $f(\epsilon)$ gives the probability that (at thermodynamic equilibrium) an electron will occupy a state having energy ϵ . Alternatively, it gives the average number of electrons that will occupy that state given the restriction imposed by the Pauli exclusion principle:

The Fermi-Dirac distribution $f(E)$ is the occupation probability of single-electron state with energy E . It is given by:

$$f(E) = \frac{1}{e^{(E - E_F)/kT} + 1}$$

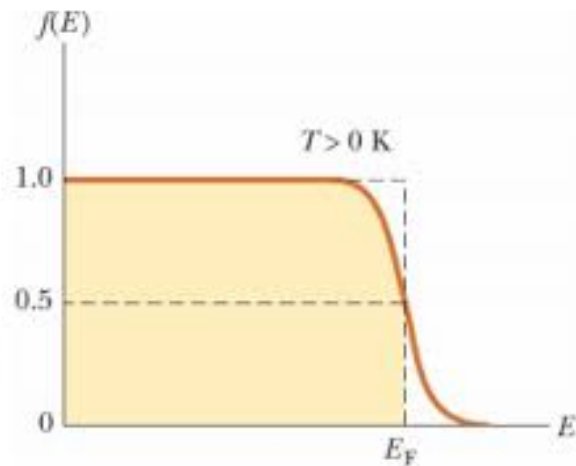
Boltzmann's constant $\rightarrow k$
 Absolute temperature $\rightarrow T$
 The +1 is a consequence of the Pauli exclusion principle: it guarantees that $f(E) \leq 1$.

Fermi function

- In thermal equilibrium, the probability of occupancy of any state is given by the Fermi function:

$$F(E) = \frac{1}{1 + e^{\frac{E - E_f}{kT}}}$$

- At the energy $E = E_f$ the probability of occupancy is $1/2$.
- At high energies, the probability of occupancy approaches zero exponentially
- At low energies, the probability of occupancy approaches 1

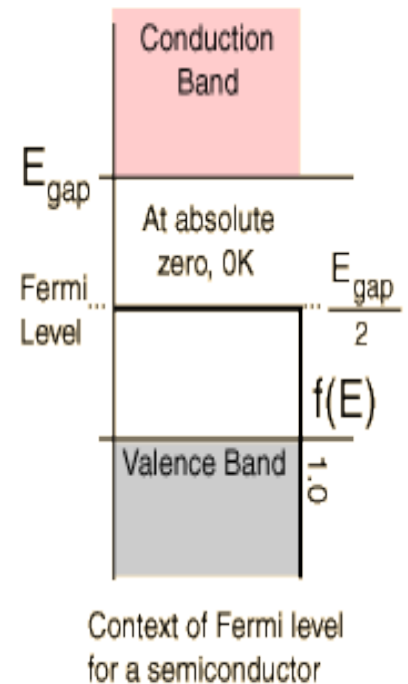


The significance of the Fermi energy is most clearly seen by setting $T=0$. At absolute zero, the probability is $=1$ for energies less than the Fermi energy and zero for energies greater than the Fermi energy. We picture all the levels up to the Fermi energy as filled, but no particle has a greater energy. This is entirely consistent with the Pauli exclusion principle where each quantum state can have one but only one particle.

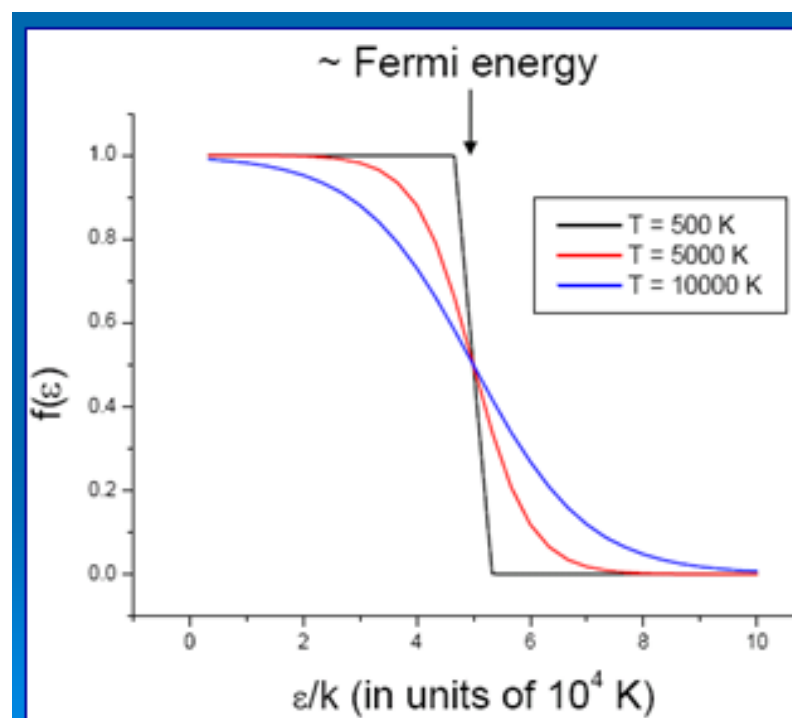
Exponential approximation (electrons)

- In semiconductors, the Fermi energy is usually in the band gap, far from either the conduction band or the valence band (compared to kT).
- For the conduction band, since the exponential is much larger than 1, we can use the approximation:

$$F(E) = \frac{1}{1 + e^{\frac{E-E_f}{kT}}} \approx \frac{1}{e^{\frac{E-E_f}{kT}}} = e^{-\left(\frac{E-E_f}{kT}\right)}$$



The changes of Fermi-Energy with Temperature



Probability of Occupation (Fermi Function) Concept

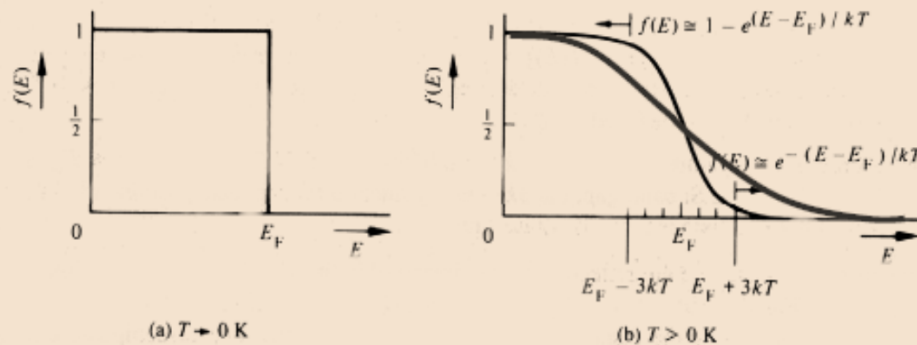


Figure 2.15 Energy dependence of the Fermi function. (a) $T \rightarrow 0$ K; (b) generalized $T > 0$ K plot with the energy coordinate expressed in kT units.

At $T=0$ K, occupancy is “digital”: No occupation of states above E_F and complete occupation of states below E_F

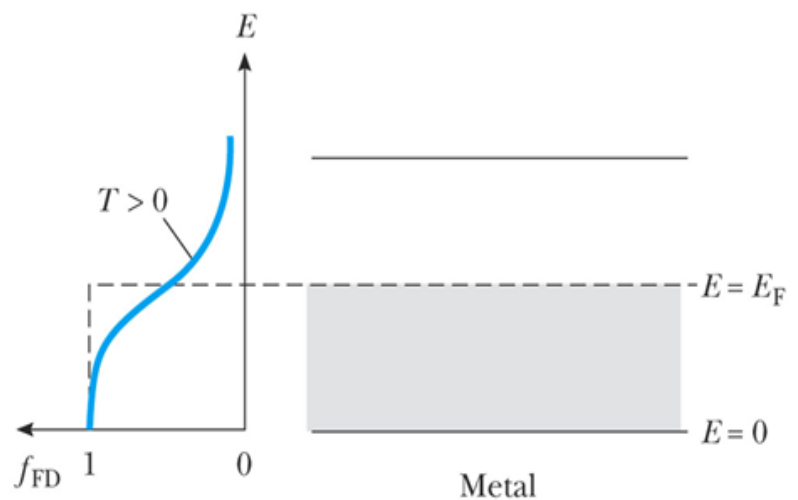
At $T>0$ K, occupation probability is reduced with increasing energy.

$f(E=E_F) = 1/2$ regardless of temperature.

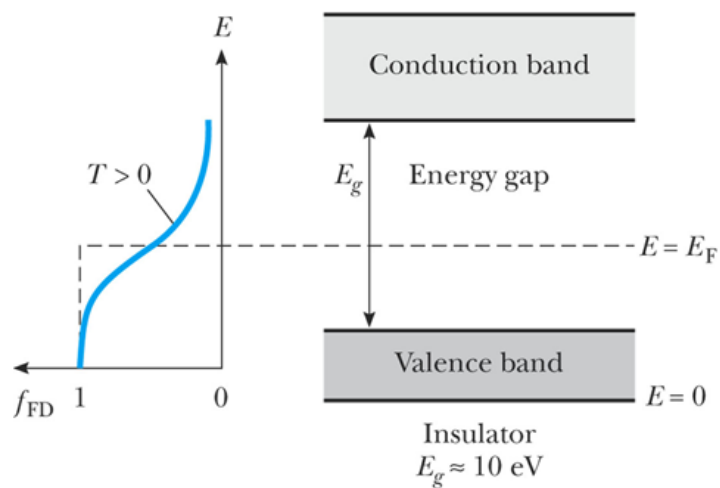
At higher temperatures, higher energy states can be occupied, leaving more lower energy states unoccupied ($1-f(E)$).

The changes of Fermi level position according the type of materials

Metal



Insulator



Semiconductor

