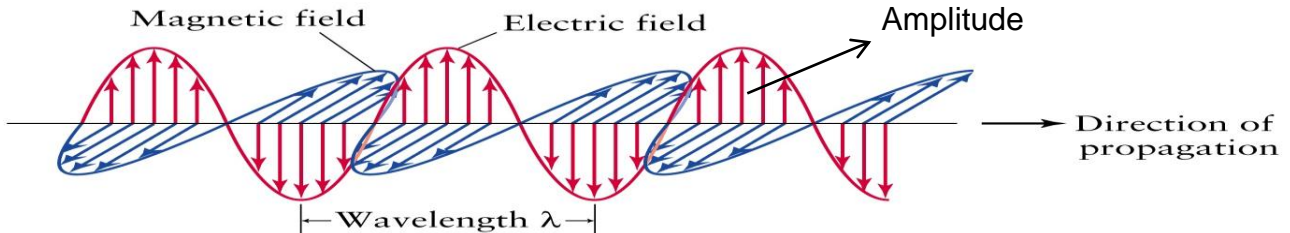


**Refrence:-**

- 1- " Antenna and Wave Propagation ", Ashish Mathur, Ranjana Trivedi & Geetike B. Mathur.
- 2- " Electromagnetic Waves and Antennas", Sophocles J. Orfanidis.
- 3- "Antenna Theory Analysis and Design", Constantine A. Balanis.
- 4- "Antennas for All Applications" 3<sup>rd</sup> edition, John D. Kraus, Ronald J. Marhefka.
- 5- "Engineering Electromagnetics", 6<sup>th</sup> Edition ,William H. Hayt, John A. Buck.

**1- Basic introduction to electromagnetic field:-**

- **Wave:-** can be defined as a motion (such as oscillation) through matter or free space.
- **An Electric field exists in the presence of a charged body.**
- **A magnetic field is a force produced by a moving electric charge that exists around a magnet or in free space.**
- **time-varying fields or waves are usually due to accelerated charges or time-varying current. Time-varying current → Electromagnetic fields (or waves)**
- **Electromagnetic (EM) radiation:-** is an electric field that travels away from some source ( like antenna, sun, radio tower...etc). Atraveling electric filed has associated magnetic field with it. Both these fields are perpendicular to each other and both are also perpendicular to the direction of wave propagation .



Fig(1) free space electromagnetic wave

**1-1 Electromagnetic waves spectrum:-**

Continuous range of EM radiation from very short wavelengths ( $<300 \times 10^{-9}$  m) (high energy) to very long wavelengths (cm, m, km) (low energy). Energy is related to wavelength (and hence frequency), EM waves characterized by:-

- **Wavelength ( $\lambda$ )** is the distance traveled by the energy during one cycle from crest to crest , usually expressed in metric units (meters, centimeters, etc.).
  - **frequency** is the number of cycles repeated during unit time (usually 1 second) . This is given in **hertz** (cycles per second). A kilohertz (kHz) is 1,000 cycles per second.
- Note:-** As frequency increases, wavelength becomes smaller. EM waves can be generated in different frequency bands: radio, microwave, infrared, visible, ultraviolet, x-rays, gamma rays.

- Amplitude (A) (m):- The amplitude of the wave is represented by the length of the electrical vector at a maximum or minimum in the wave.
- Velocity(v or C):- The velocity of a wave is defined as the multiplication of the frequency times the wavelength. This means:-

$$C=V = f \lambda \quad \text{in free space (vacuum)} =V=C=3 \times 10^8 \text{ m/sec}$$

One of the great discoveries in the history of electromagnetism is that electromagnetic waves travel at the speed of light (The velocity of light in vacuum is greater than its velocity in any other medium).

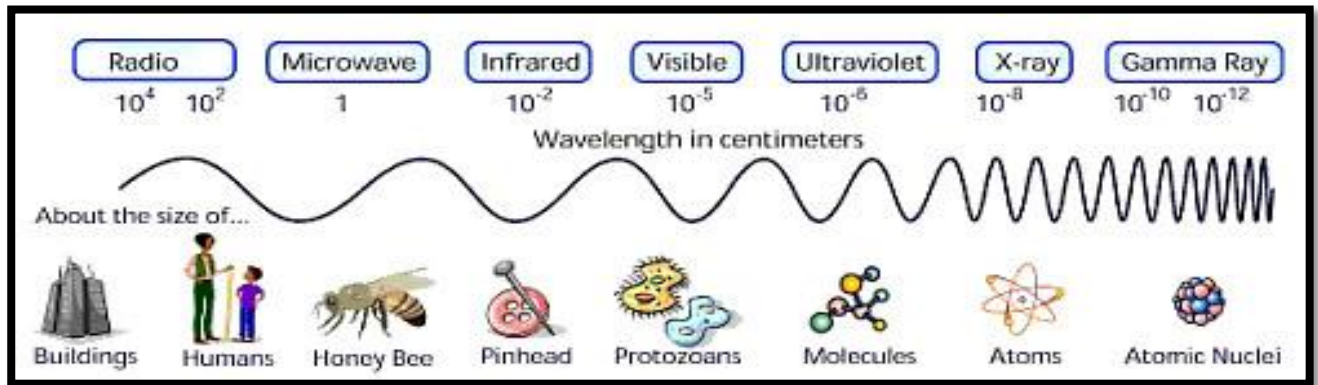


Fig.(2) electromagnetic Spectrum.

Band Abbreviation	Range of frequency	Range of wavelength
Audio frequency AF	20 to 20,000 Hz	15,000,000 to 15,000 m
Radio frequency RF	10 kHz to 300,000 MHz	30,000 m to 0.1 cm
Very low frequency VLF	10 to 30 kHz	30,000 to 10,000 m
Low frequency LF	30 to 300 kHz	10,000 to 1,000 m
Medium frequency MF	300 to 3,000 kHz	1,000 to 100 m
High frequency HF	3 to 30 MHz	100 to 10 m
Very high frequency VHF	30 to 300 MHz	10 to 1 m
Ultra high frequency UHF	300 to 3,000 MHz	100 to 10 cm
Super high frequency SHF	3,000 to 30,000 MHz	10 to 1 cm
Extremely high frequency EHF	30,000 to 300,000 MHz	1 to 0.1 cm
Heat and infrared*	$10^6$ to $3.9 \times 10^8$ MHz	0.03 to $7.6 \times 10^{-5}$ cm
Visible spectrum*	$3.9 \times 10^8$ to $7.9 \times 10^8$ MHz	$7.6 \times 10^{-5}$ to $3.8 \times 10^{-5}$ cm
Ultraviolet*	$7.9 \times 10^8$ to $2.3 \times 10^{10}$ MHz	$3.8 \times 10^{-5}$ to $1.3 \times 10^{-6}$ cm
X-rays*	$2.0 \times 10^9$ to $3.0 \times 10^{13}$ MHz	$1.5 \times 10^{-5}$ to $1.0 \times 10^{-9}$ cm
Gamma rays*	$2.3 \times 10^{12}$ to $3.0 \times 10^{14}$ MHz	$1.3 \times 10^{-8}$ to $1.0 \times 10^{-10}$ cm
Cosmic rays*	$>4.8 \times 10^{15}$ MHz	$<6.2 \times 10^{-12}$ cm

Table (1) Electromagnetic spectrum. (\* value approximate)

### 1-2 Radio frequency (RF)

any of the electromagnetic wave frequencies that lie in the range extending from below 3 kilohertz to about 300 gigahertz and that include the frequencies used for communications signals (as for radio and television broadcasting and cell-phone and satellite transmissions) or radar signals.( RF have the **longest** wavelengths and **lowest** frequencies of all the electromagnetic waves).

**Radio propagation** :- is the behavior of radio waves when they are transmitted, or propagated from one point on the earth to another, or into various parts of the

atmosphere. Like light waves, radio waves are affected by the phenomena of reflection, refraction, diffraction, absorption, polarization and scattering.

Band name	Abbreviation	Frequency and wavelength in air	Example uses
Tremendously low frequency	TLF	<3Hz > 100,000 km	الضوء الكهرومغناطيسية الطبيعية والصناعية
Extremely low frequency	ELF	3–30Hz 100,000 km – 10,000 km	اتصالات الغواصات ويستخدم أيضا في القدرة الكهربائية المتناوبة (٥٠-٦٠) هرتز والترددات السمعية
Super low frequency	SLF	30–300Hz 10,000 km – 1000 km	اتصالات الغواصات
Ultra low frequency	ULF	300–3000Hz 1000 km – 100 km	اتصالات الغواصات واتصالات المناجم
Very low frequency	VLF	3–30kHz 100 km – 10 km	الملاحة، اشارات ضبط الوقت، مراقبة معدل نبضات القلب لاسلكيا، محطات التلغراف لمسافات بعيدة وفي علوم الارض
Low frequency	LF	30–300kHz 10 km – 1 km	المشاعل الملاحية، اشارات ضبط الوقت والموجات الطويلة (اي ام) للإذاعة البرامج في اوريا وبعض اجزاء اسيا
Medium frequency	MF	300–3000kHz 1 km – 100 m	موجة (اي ام) متوسطة المدى للإذاعة، راديو هاواي، وخدمات سفن الشواطئ.
High frequency	HF	3–30MHz 100 m – 10 m	في الموجات القصيرة وترددات الراديو وكذلك حزمة سترن وخدمات الاتصالات من نقطة الى اخرى ولمسافات بعيدة ومتوسطة ويستخدم في الإذاعة.
Very high frequency	VHF	30–300 MHz 10 m – 1 m	في حزمة اف ام وكذلك التلفزيون والرادار
Ultra high frequency	UHF	300–3000 MHz 1 m – 100 mm	في التلفزيون، الافران المايكروية، الهواتف الجوالة، شبكة اللاسلكي المحلية (الوايرليس) وبلوتوث.
Super high frequency	SHF	3–30 GHz 100 mm – 10 mm	علم الفلك، الرادار، اتصالات الاقمار الصناعية، وترددات المايكرويف، شبكة الوايرليس
Extremely high frequency	EHF	30–300 GHz 10 mm – 1 mm	علم الفلك، ترددات المايكرويف العالية، توجيه الاسلحة، ماسح ضوئي
Terahertz or Tremendously high frequency	THz or THF	300–3,000 GHz 1 mm – 100 μm	التصوير البديل للاشعة السينية في بعض التطبيقات الطبية، المتحسسات عن بعد وانظمة الاسلحة المتقدمة والترددات المايكرويف العالية

Table (2) radio wave spectrum.

### 1-3 The electrical properties of the transmission medium:-

The transmission media that are used to convey information can be classified as guided or unguided. Guided media provide a physical path along which the signals are propagated; these include twisted pair, coaxial cable, and optical fiber. Unguided media employ an antenna for transmitting through air, vacuum, or water, then the electrical properties of the transmission medium

- a- Permittivity ( $\epsilon$ )** is the measure of the resistance that is encountered when forming an electric field in a medium. In other words, permittivity is a measure of how an electric field affects, and is affected by, a dielectric medium. The permittivity of a medium describes how much electric field (more correctly, flux) is 'generated' per unit charge in that medium. More electric flux exists in a

medium with a high permittivity (per unit charge) . each material has certain constant limits the value of permittivity and the permittivity is equal to:-

$$\epsilon = \epsilon_0 \epsilon_r$$

Where:-  $\epsilon_0 = 8.8541878176.. \times 10^{-12}$  F/m is the vacuum permittivity.

$\epsilon_r$  is the relative permittivity of the material. This value is different from medium to anther and is equal to one in free space.

**b- Permeability ( $\mu$ )** also called magnetic permeability, is a constant of proportionality that exists between magnetic induction and magnetic field intensity. The permeability is equal to:

$$\mu = \mu_0 \mu_r$$

Where:-

$\mu_0$  is absolute permeability and is equal to  $4\pi \times 10^{-7}$  H/m.

$\mu_r$  is the relative permeability of the material. This value is different from medium to anther and is equal to one in free space.

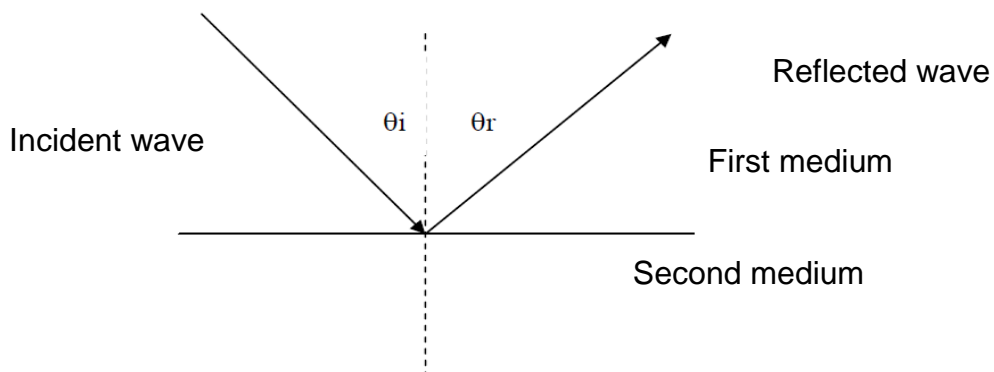
**c- Conductivity ( $\sigma$ )** a measure of a material's ability to conduct an electric current. Perfect dielectric has the conductivity is equal to zero approximately.

#### 1-4 Optical properties of electromagnetic waves:-

##### **a- Reflection (bouncing)**

When a radio wave propagating in one medium impinges upon another medium having different electromagnetic properties, it will be partially reflected back into the first medium, and partially transmitted (refracted) into the second medium. The basic property of reflection is that the direction of the reflected wave is symmetric to the direction of the incident wave with respect to the surface normal. The rule for reflection is simply stated as:

$$\theta_i = \theta_r$$



Fig(3) Reflected wave

**b- Refraction (Banding)**

Refraction occurs when waves pass from one density medium to another with different velocities of propagation. The amount of bending or refraction that occurs at the interface of two materials of different densities is quite predictable and depends on the refractive index which is simply the ratio of the velocity of propagation in a given material (EM wave propagate at the speed of light in a vacuum, in other mediums like air or glass, the speed of propagation is slower).

$$n = \frac{\text{speed of light in vacumm}}{\text{speed of light in other medium}} = \frac{c}{v_p}$$

The Snell's law connect the relation between the incident wave in the 1<sup>st</sup> medium and the refracted wave in the second medium

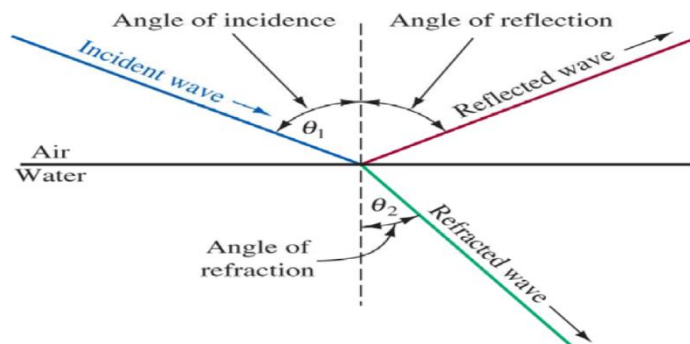
$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$n_1$  = refractive index of first medium.

$n_2$  = refractive index of second medium.

$\theta_1$  = the incident angle.

$\theta_2$  = the refracted angle.



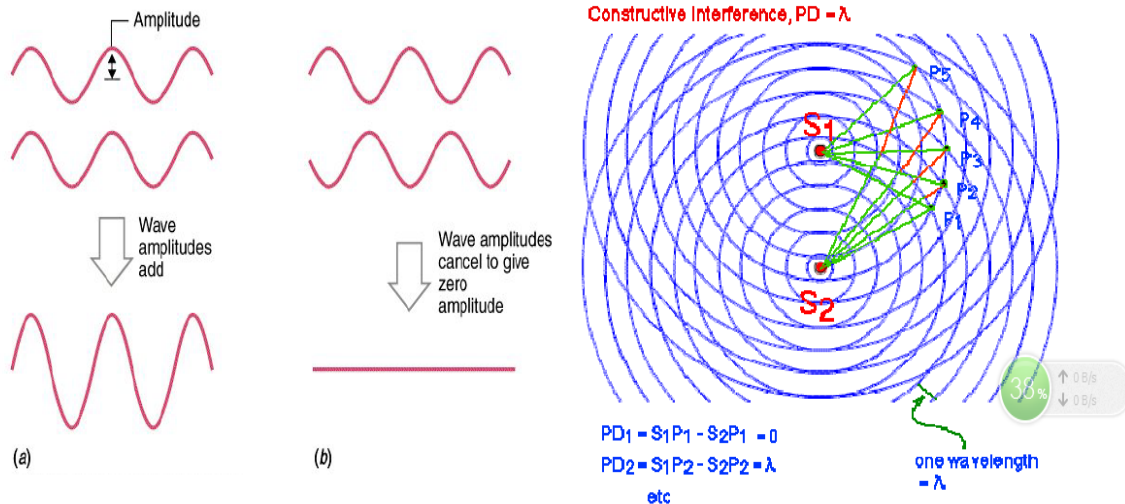
Note:- if the incident medium has a lower index of refraction then the reflected wave has a 180° phase shift upon reflection. Conversely, if the if the incident medium has a large index of refraction then the reflected wave has no phase shift.

**c- Diffraction (scattring)**

Scattering (diffraction) is a general interaction process between electromagnetic waves and various objects. Small and irregularly shaped objects, such as trees, lamp posts, street signs or irregularities in building walls are considered as scatterers. The building materials (steel, wire meshes, and plasterboards) cause scattering that in turn causes signal energy to be reradiated in many different directions.

**d- Interference (colliding)**

Radio wave interference occurs when two or more electromagentic waves combine in such away to occupy the same position in space using the principle of linear superposition.



**e- Absorption**

is the loss of signal power as the signal moves through the transmission medium, and that reduces the available bandwidth. The following two materials are considered the largest absorption for microwaves:-

- 1- Metals: the electrons in the metals can moving freely through the metal, and then these electrons can swinging and absorbed the electromagnetic waves that pass through the metal.
- 2- Water: the electromagnetic waves in the water causes jostling the particulars of the water about the electromagnetic waves, and then result in absorption the energy of these waves.

**2- Time varying Maxwell's Equation's:-**

Maxwell's equations describe all (classical) electromagnetic phenomena:-

Differential (or Point) Form	Integral Form	Remarks
$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$	$\oint E \cdot dL = -\frac{\partial}{\partial t} \int B \cdot dS$	Faraday's Law
$\nabla \times \bar{H} = \bar{J}_T + \frac{\partial \bar{D}}{\partial t}$	$\oint H \cdot dL = \int J \cdot dS$	Amper's Law
$\nabla \times \bar{D} = \rho_v$	$\oint D \cdot dS = \int \rho_v \cdot dv$	Gauss's Law for electric field
$\nabla \times \bar{B} = 0$	$\oint B \cdot dS = 0$	Gauss's Law for magnetic field

Where:-

H = magnetic field (A/m).	E = electric field (V/m).
B = magnetic flux density(wb/m <sup>2</sup> ) or Tesla , B=μH	D = electric flux density, (C/m <sup>2</sup> ) = εE
(∂B/∂t) = time-derivative of magnetic flux density	(∂D/∂t) = displacement electric current density (A/m <sup>2</sup> )=J <sub>d</sub> .

$\rho_v$ = volume charge density (C/m <sup>3</sup> )
$J_T$ = conduction current density (A/m <sup>2</sup> ) = $J_{cond} + J_{Sou} = \sigma E + J$ , $J$ = source current

**1- Faraday's Law:-**

Michael Faraday discovered that the induced  $V_{emf}$  (in volts), in any closed circuit is equal to the time rate of change of the magnetic flux linkage by the circuit:-

$$e.m.f = -\frac{d\psi}{dt} \text{ [lenz's law]} \quad \text{or} \quad e.m.f = -N \frac{d\psi}{dt} \quad (1)$$

➤ Lenz's rule gives the direction of the induced emf which states that the induced current produced in a circuit always in such a direction that it opposes the change or the cause that produces it.

$\psi$  is the Magnetic Flux within a circuit.

**e.m.f** is the electro-motive force, which is basically a voltage source.

The total magnetic flux is simply the integral (or sum) of the **B** field over the area enclosed by the wire:

$$\psi = \int_s \mathbf{B} \cdot d\mathbf{s} \quad (2)$$

$$v = \int E \cdot dL \quad (3)$$

$$E = \frac{dv}{dL} \quad (4)$$

$$e.m.f_{total} = \oint E \cdot dL \quad (5)$$

From stokes theorem which can be used to transform the line integral to the surface integral of curl:-

$$\oint E \cdot dL = \int \nabla \times E \cdot ds \quad \text{[stokes theorem]} \quad (6)$$

substituting equations (2)& (6) in equation (1):-

$$\int \nabla \times E \cdot ds = -\frac{d}{dt} \int B \cdot ds \quad (7)$$

It follows that:-

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (8)$$

Which is the differential form of Faraday's Law.

**2- Amper's Law:-**

For static EM fields, we recall that:-

$$\nabla \times \mathbf{H} = \mathbf{J} \quad (1)$$

But the divergence of the curl of any vector field is identically zero:

$$\nabla \cdot (\nabla \times \mathbf{H}) = 0 = \nabla \cdot \mathbf{J} \quad (2)$$

The continuity of current requires that:

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t} \neq 0 \quad (3)$$

Thus eqs. 2 and 3 are obviously incompatible for time-varying conditions. We must modify eq. 1 to agree with eq. 3. To do this, we add a term to eq. 1, so that it becomes:

$$\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{J}_d \quad (4)$$

where  $\mathbf{J}_d$  is to be determined and defined. Again, the divergence of the curl of any vector is zero. Hence:

$$\nabla \cdot (\nabla \times \mathbf{H}) = 0 = \nabla \cdot \mathbf{J} + \nabla \cdot \mathbf{J}_d \quad (5)$$

In order for eq. 5 to agree with eq. 3:

$$\nabla \cdot \mathbf{J}_d = -\nabla \cdot \mathbf{J} \rightarrow \nabla \cdot \mathbf{J}_d = \frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial t} (\nabla \cdot \mathbf{D}) = \nabla \cdot \frac{\partial \mathbf{D}}{\partial t} \quad (6)$$

or 
$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t}$$

Substituting eq. 6 into eq. 3 results in:

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

This is Maxwell's equation (based on Ampere's circuit law) for a time-varying field. The term  $\mathbf{J}_d = \partial \mathbf{D} / \partial t$  is known as *displacement current density* and  $\mathbf{J}$  is the conduction and source current density ( $\mathbf{J} = \sigma \mathbf{E} + \mathbf{J}_d$ ).

### 3- Poynting Vector and Flow of Power:-

Poynting was the developer and eponym of the Poynting vector, which describes the direction and magnitude of electromagnetic energy flow and is used in the Poynting theorem, a statement about energy conservation for electric and magnetic fields, This relation can be obtained from Maxwell's equation as follows:-

$$\nabla \times \bar{\mathbf{E}} = -\frac{\partial \bar{\mathbf{B}}}{\partial t} \quad \text{and} \quad \nabla \times \bar{\mathbf{H}} = \bar{\mathbf{J}} + \frac{\partial \bar{\mathbf{D}}}{\partial t} \quad (1)$$

Multiply the above equations by dot  $\bar{\mathbf{E}}$  and dot  $\bar{\mathbf{H}}$ , from these we obtain:-

$$\bar{\mathbf{H}} \cdot (\nabla \times \bar{\mathbf{E}}) = -\bar{\mathbf{H}} \cdot \frac{\partial \bar{\mathbf{B}}}{\partial t} \quad \text{and} \quad \bar{\mathbf{E}} \cdot (\nabla \times \bar{\mathbf{H}}) = \bar{\mathbf{J}} \cdot \bar{\mathbf{E}} + \bar{\mathbf{E}} \cdot \frac{\partial \bar{\mathbf{D}}}{\partial t} \quad (2)$$

Subtract following vector identity:-

$$\bar{\mathbf{H}} \cdot (\nabla \times \bar{\mathbf{E}}) - \bar{\mathbf{E}} \cdot (\nabla \times \bar{\mathbf{H}}) = \nabla \cdot (\bar{\mathbf{E}} \times \bar{\mathbf{H}}) \quad (3)$$

We then have:-

$$\nabla \cdot (\bar{\mathbf{E}} \times \bar{\mathbf{H}}) = -\bar{\mathbf{J}} \cdot \bar{\mathbf{E}} - \bar{\mathbf{H}} \cdot \frac{\partial \bar{\mathbf{B}}}{\partial t} - \bar{\mathbf{E}} \cdot \frac{\partial \bar{\mathbf{D}}}{\partial t} \quad (4)$$

Next, assume that Ohm's Law applies for the electric current:-

$$\mathbf{J} = \sigma \bar{\mathbf{E}} \quad (5)$$

$$\nabla \cdot (\bar{\mathbf{E}} \times \bar{\mathbf{H}}) = -\sigma (\bar{\mathbf{E}} \cdot \bar{\mathbf{E}}) - \bar{\mathbf{H}} \cdot \frac{\partial \bar{\mathbf{B}}}{\partial t} - \bar{\mathbf{E}} \cdot \frac{\partial \bar{\mathbf{D}}}{\partial t} \quad (6(a))$$

$$\text{or } \nabla \cdot (\bar{\mathbf{E}} \times \bar{\mathbf{H}}) = -\sigma |E|^2 - \bar{\mathbf{H}} \cdot \frac{\partial \bar{\mathbf{B}}}{\partial t} - \bar{\mathbf{E}} \cdot \frac{\partial \bar{\mathbf{D}}}{\partial t} \quad (6(b))$$



From calculus (chain rule), we have that:-

$$\vec{E} \cdot \frac{\partial \vec{D}}{\partial t} = \epsilon \left( \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \right) = \epsilon \frac{1}{2} \frac{\partial}{\partial t} (\vec{E} \cdot \vec{E}) \text{ and } \vec{H} \cdot \frac{\partial \vec{B}}{\partial t} = \mu \left( \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} \right) = \mu \frac{1}{2} \frac{\partial}{\partial t} (\vec{H} \cdot \vec{H}) \quad (7)$$

Hence, we have;-

$$\nabla \cdot (\vec{E} \times \vec{H}) = -\sigma |E|^2 - \mu \frac{1}{2} \frac{\partial}{\partial t} (\vec{H} \cdot \vec{H}) - \epsilon \frac{1}{2} \frac{\partial}{\partial t} (\vec{E} \cdot \vec{E}) \quad (8(a))$$

Final differential (point) form of the Poynting theorem:

$$\nabla \cdot (\vec{E} \times \vec{H}) = -\sigma |E|^2 - \frac{\partial}{\partial t} \left( \frac{1}{2} \mu |H|^2 \right) - \frac{\partial}{\partial t} \left( \frac{1}{2} \epsilon |E|^2 \right) \quad (8(b))$$

Volume (integral) form (over volume V):-

$$\int \nabla \cdot (\vec{E} \times \vec{H}) \cdot dv = - \int \sigma |E|^2 \cdot dv - \int \frac{\partial}{\partial t} \left( \frac{1}{2} \mu |H|^2 \right) \cdot dv - \int \frac{\partial}{\partial t} \left( \frac{1}{2} \epsilon |E|^2 \right) \quad (9)$$

Using divergence theorem, the last term can be change from avolume integral to a surface integral over the surface S surrounding V, then Final volume form of Poynting theorem:-

$$\oint (\vec{E} \times \vec{H}) \cdot ds = - \int \sigma |E|^2 \cdot dv - \int \frac{\partial}{\partial t} \left( \frac{1}{2} \mu |H|^2 \right) \cdot dv - \int \frac{\partial}{\partial t} \left( \frac{1}{2} \epsilon |E|^2 \right) \cdot dv \quad (10)$$

For a stationary surface:

$$\oint (\vec{E} \times \vec{H}) \cdot ds = - \int \sigma |E|^2 \cdot dv - \frac{\partial}{\partial t} \int \left( \frac{1}{2} \mu |H|^2 \right) \cdot dv - \frac{\partial}{\partial t} \int \left( \frac{1}{2} \epsilon |E|^2 \right) \cdot dv \quad (11)$$

Power dissipation as heat (Joule's law)

Rate of change of stored magnetic energy

Rate of change of stored electric energy

Right-hand side = power flowing into the volume of space.

$\oint (\vec{E} \times \vec{H}) \cdot ds$  =power flowing out of the region.

$-\oint (\vec{E} \times \vec{H}) \cdot ds$  =power flowing into the region.

Define the Poynting vector:  $\vec{S} = \vec{E} \times \vec{H}$  W/m<sup>2</sup>.

The Vector  $\mathbf{S} = \mathbf{E} \times \mathbf{H}$  has interpreted as representing the amount of field energy passing through the unit area of surface in unit time normally to the direction of flow of energy. This statement is termed as *Poynting's theorem* and the vector S is called *Poynting Vector*. The direction of flow of energy is perpendicular to vectors  $\mathbf{E}$  and  $\mathbf{H}$

#### 4- Instantaneous, Average, and Complex Poynting vector:-

The time-dependent power flow density of an electromagnetic wave is given by the instantaneous Poynting vector

$$\vec{S}_{inst} = \vec{E} \times \vec{H}$$

Consider time-harmonic fields represented in terms of their phasors:-

$$\vec{S}_{inst} = \vec{E} \times \vec{H} \quad (1)$$

$$E(t) = \text{Re}(E e^{j\omega t}) = \frac{1}{2}[E e^{j\omega t} + E^* e^{-j\omega t}] \quad (2a)$$

$$H(t) = \text{Re}(H e^{j\omega t}) = \frac{1}{2}[H e^{j\omega t} + H^* e^{-j\omega t}] \quad (2b)$$

$$\vec{S}_{inst} = \frac{1}{4}[E \times H^* + E^* \times H] + \frac{1}{4}[E \times H e^{j2\omega t} + E^* \times H^* e^{-j2\omega t}] \quad (3)$$

Let :-

$$M = E \times H^* \quad \text{and} \quad M^* = (E \times H^*)^* = E^* \times H$$

$$N = E \times H \quad \text{and} \quad N^* = (E \times H)^* = E^* \times H^*$$

Then :-

$$\vec{S}_{inst} = \frac{1}{4}[M + M^*] + \frac{1}{4}[N e^{j2\omega t} + N^* e^{-j2\omega t}] \quad (4)$$

$$\vec{S}_{inst} = \frac{1}{2}\text{Re}(M) + \frac{1}{2}\text{Re}[N e^{j2\omega t}] \quad (5)$$

$$S_{ave} = \frac{1}{T} \int_0^T S_{inst} dt \quad (6)$$

$$S_{ave} = \frac{1}{T} \int_0^T \frac{1}{2} \text{Re}(E \times H^*) dt + \frac{1}{T} \int_0^T \frac{1}{2} \text{Re}(E \times H) e^{j2\omega t} dt \quad (7)$$

$$S_{ave} = \frac{1}{2} \text{Re}(E \times H^*) + \frac{1}{2T} \text{Re}(E \times H) \int_0^T \cos(2\omega t) dt \quad (8)$$

$$S_{ave} = \frac{1}{2} \text{Re}(E \times H^*) \quad (9)$$

$$S_{ave} = \frac{1}{2} \text{Re}(S_{inst}) \quad (10)$$

Where:-

$$S_{com} = \frac{1}{2}(E \times H^*)$$

$S_{com}$  is the complex poynting vector.