Realizations of Digital Filters

Before taking examples about the Realization, we need to get an overview about the Frequency response of LTI processors (it is also a Digital filters).

The alternative way to find the frequency response of a digital processors (digital filter) is by its difference equation.

\[ \sum_{k=0}^{N} a_k y(n - k) = \sum_{k=0}^{M} b_k x(n - k) \]

Taking the Fourier transform of both sides:

\[ \sum_{k=0}^{N} a_k \exp(-jkw)Y(w) = \sum_{k=0}^{M} b_k \exp(-jkw)X(w) \]

The result directly from the linearity and time shifting properties of the transform, now:

\[ Y(w) = X(w)H(w) \]
\[ H(w) = \frac{Y(w)}{X(w)} = \frac{\sum_{k=0}^{M} bk \exp(-jkw)}{\sum_{k=0}^{N} ak \exp(-jkw)} \]

\[ H(w) = \frac{b0 + b1 \exp(-jw) + b2 \exp(-2jw).....}{a0 + a1 \exp(-jw) + a2 \exp(-2jw).....} \]

**Example:** Find the frequency response \( H(w) \) of the system shown below and sketch its characteristics over the range \( 0 \leq w \leq \pi \).

**Ans:**
Example: A system with difference equation $y(n)=1.5y(n-1)-0.85y(n-2)+x(n)$. Find its frequency response and draw its characteristics over the range $0 \leq \omega \leq \pi$. 

Ans:
Example: Draw the Direct Form I realization of a system has impulse response $h(n) = \{1, 2, 3, 3, 2\}$. 

Ans:
Example: Draw the direct-form I realization of an IIR filter with the following transfer function.

\[ H(z) = \frac{1 + z^{-1} + z^{-2} + z^{-3}}{1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}} \]

Ans:
Example: A system has transfer function:

\[ H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 4z^{-1} + 11z^{-2} - 2z^{-3}}{1 - 5/4z^{-1} + 3/4z^{-2} - 1/8z^{-3}} \]

Draw the realization using:

1. Direct-Form I.
2. Direct-Form II.

Ans:
Example: Find the Cascade Realization in term of quadratic sections for the following transfer function:

\[ H(z) = \frac{Y(z)}{X(z)} = \frac{8Z^3 - 4Z^2 + 11Z - 2}{(z - 0.25)(Z^2 - Z + 0.5)} \]

Ans: