Module 2

Fuzzy Control Systems Design

Module 2 Objectives

- To do a brief review on basics of control.
- To understand the need for fuzzy logic control.
- To understand how to design a fuzzy logic controller.
- Understanding the components of a fuzzy logic controller.
- To understand how inferencing is done in a fuzzy logic controller.
- To be able to understand how rules can be developed in a fuzzy logic controller.
- To study several types of defuzzification techniques.
- To study the design procedure of a fuzzy logic controller.
- At the end of the module, the student should be able to understand the basic components of a fuzzy logic controller and how they function.
2.1 Overview of Fuzzy Logic Control

2.1.1 Review of Control System Basics

2.1.2 Why fuzzy logic control?

2.1.3 Configuration of a fuzzy logic control system
2.1.1 Review of Control System Basics

Example of control systems

- Cars, Aeroplanes, Trains, Ships, ...
- Nuclear Plants, Power Stations, ...
- Washing Machines, Refrigerators, ...
- Disk Drives, Stepper Motors, ......
- Chemical Processes, Food Processing Plants ......
- Robotic Manipulators, NC Machines, ...

Example of Control Variables

- Position
- Speed
- Liquid level
- Temperature
- Pressure
- Turgidity
- Concentration
- Intensity
- Etc.

Control objective

- In many control systems, the objective is to design a controller such that output can be controlled by giving the desired signals at the input.

Example of control

- Alignment of the front wheel of a vehicle follows that of the driver’s steering wheel
- Rudders of a boat follow its steering wheel
- Concentration in liquids follows a given set-point
- Temperature of furnace follows a given set-point
Control Systems Performance

• What are the factors affecting control systems?
  – Choice of the Controllers (Algorithms)
  – Tuning of Controllers
  – Selection of Control Variables
  – Selection of Performance Index
  – Types of Sensors (Measurement Errors)
  – Actuators
  – Disturbances/Environment
  – Noise
  – Control Configuration
  – Design Techniques
  – Nonlinearity

• Several performance indices that can be used to measure control system performance
  – Integral Squared Error (ISE)
  – Integral Absolute Error (IAE)
  – Integral Time Absolute Error (ITAE)
  – Integral Time Squared Error (ITSE)
  – Minimum Fuel, Time, etc.
Control system components

- A control system is made up of many sub-control systems and components.

A space shuttle

A military helicopter

Analysis in Time Domain

- By observing the system time response we can measure its performance.
- To measure the swiftness of system response: Check rise-time and peak-time.
- To measure the closeness of system response to the desired response: check overshoot, settling-time, rise-time, steady-state error, ISE, etc.
- Other analysis can also be done.
Example of Control Paradigms

Control Design Techniques and Control Paradigms

- Classical
  - Root Locus
  - Bode Plot
  - Nichols Chart
  - Nyquist Plots

- Modern
  - State Feedback
  - State Estimation
  - Observers
  - Optimal Control
  - Robust Control
  - H-Infinity
  - Internal Model Control
  - Adaptive Control

- Artificial Intelligence
  - Neuro-Control
  - Fuzzy Control
  - Genetic Algorithms
  - Knowledge Based Systems

Example of Controllers that can be applied in control systems

Conventional Controllers
- Bang-Bang (on-off)
- Proportional
- PID Controller

Adaptive Controllers
- Self-Tuning Controller
- Self-Tuning PID Controller
- Auto-Tuning PID Controller
- Pole Placement

Predictive Controllers
- Smith Predictor
- Generalized Predictive Controllers

Intelligent Controllers
- Knowledge Based
- Fuzzy Logic Controllers
- Neuro-Controllers
- Adaptive Fuzzy
2.1.2 Why Fuzzy Control?

- During the past decade, fuzzy logic control has emerged as one of the most active and fruitful research areas in the application of fuzzy set theory, fuzzy logic and fuzzy reasoning.

- The idea was first proposed by Mamdani and Assilian around 1972.

- Many industrial and consumer products using fuzzy logic technology have been built and successfully sold worldwide.

- In contrast to conventional control techniques, fuzzy logic control is best utilized in complex ill-defined processes that can be controlled by a skilled human operator without much knowledge of their underlying dynamics.
• The basic idea behind fuzzy logic control is to incorporate the “expert experience” of a human operator in the design of a controller in controlling a process whose input-output relationship is described by a collection of fuzzy control rules (e.g. IF-THEN rules) involving linguistic variables.

• This utilization of linguistic variables, fuzzy control rules, and approximate reasoning provides a means to incorporate human expert experience in designing the controller.

Benefits of Fuzzy Control

• Suitable for Complex Ill-defined Systems where mathematical modeling is difficult or Plant is too abstract e.g. Complex chemical plant, cement-kiln, etc.

• Able to design along linguistic lines – usage of rules based on experience

• Better performance than Conventional PID Controllers

• Simple to design
### 2.2 Designing a Fuzzy Logic Control System

- Where do we start?
- What are the Basics/Background needed?
- What are the components of a Fuzzy Controller?
- Where can it be applied?

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A fuzzy logic control system
2.2.1 Components of the fuzzy logic controller

- It generally comprises four principal components:
  - fuzzifier
  - knowledge base
  - inference engine
  - defuzzifier

- If the output from the defuzzifier is not a control action for a process, then the system is a fuzzy logic decision support system.

- The fuzzy controller itself is normally a two-input and a single-output component. It is usually a MISO system.
Design Process

- Identify controller Inputs and Outputs as the Fuzzy variables
- Break up Inputs and Outputs into several Fuzzy Sets and label them according to the problem to be solved and set up the fuzzy variables on the appropriate universes of discourse

<FUZZIFICATION>

- Configure/develop RULES to solve the problem
- Choose Inference Encoding procedure
- Choose a DEFUZZIFICATION Strategy
- Tune the adjustable parameters

Discuss how to design a fuzzy controller for the following process?

- Hot shower system
- Automatic Fuzzy Car Wipers
- Rice Cooker
- Vacuum Cleaner
- Air conditioner
- Water Bath Temperature Control System
2.2.2 Operations of the fuzzy controller

- A fuzzy logic controller usually has 2 inputs and 1 output for a SISO plant.

- The 2 inputs are used for accessing the current conditions of the process such that the necessary action can be taken by providing the correct control signal (which is its output).

- The inputs to the fuzzy controller is usually the error (e) which measures the system performance and the rate at which the error changes (Δe). and the output is the change of the control signal (Δu).

![Diagram of the fuzzy logic controller's components]

Components of the Fuzzy Logic Controller

- Fuzzification
- Inference Mechanism
- Defuzzification
• A fuzzifier then transforms the crisp values of e and $\Delta e$ into corresponding fuzzy values (usually there are several fuzzy values of e and $\Delta e$).

• From the rule base, the fuzzy values of e and $\Delta e$ determine which rules are to be fired through an inferencing algorithm.

• Several values of $\Delta u$ will then be obtained and a defuzzification mechanism will then transform these values into one crisp value.

• The actual control signal is obtained by adding $\Delta u$ to the past value of $u$ which is send to the plant.

2.2.2 Fuzzification

What is Fuzzification?
Fuzzification

- Involves the conversion of the input/output signals into a number of fuzzy represented values (fuzzy sets).
- Choose an appropriate Membership Function to represent each Fuzzy Set
- Label the Fuzzy Sets appropriately such that they reflect the problem to be solved
- Set up the Fuzzy Sets on appropriate Universes of Discourse
- Adjust / tune the widths and centerpoints of membership functions judiciously

Consider a Fuzzy Rice Cooker

- First, we need to identify the input and output variables of the process, in this case, the rice cooker.
- Study the control mechanism
Input and Output Variables of the Fuzzy Rice Cooker

What is the output of the Fuzzy Controller?

Rate of Heating, $\Delta E$

2 Best Candidates of the Input Variables (in order to make the output decision) would be:

Amount of Rice, $W$

Temp. of Rice, $\theta$

Fuzzification

Rate of Heating, $\Delta E$

Amount of Rice, $W$

Temp. of Rice, $\theta$

- For each Variable, how many fuzzy sets should each be partitioned into?
- For simple problems, we can start with 3, usually 5.
- What would be the correct labels for each fuzzy set?
- Use appropriate membership functions, usually triangular.
Fuzzification

Rate of Heating, $\Delta E$

Membership Value, $\mu_{\Delta E}$

LOW  MEDIUM  HIGH

Amount of Rice, $W$

Membership Value, $\mu_W$

LOW  MEDIUM  HIGH

Temp. of Rice, $\theta$

Membership Value, $\mu_\theta$

LOW  MEDIUM  HIGH
Example of Standard Labels used for Quantization into 5 Fuzzy Sets (usually for Control systems)

<table>
<thead>
<tr>
<th>Label</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative Medium</td>
<td>NM</td>
</tr>
<tr>
<td>Negative Small</td>
<td>NS</td>
</tr>
<tr>
<td>Zero</td>
<td>ZE</td>
</tr>
<tr>
<td>Positive Small</td>
<td>PS</td>
</tr>
<tr>
<td>Positive Medium</td>
<td>PM</td>
</tr>
</tbody>
</table>

Consider an Air Conditioner

How do you design the fuzzification component of the fuzzy Air-conditioner system?
SOME ANALYSIS ON FUZZY SETS IN CONTROL

• Usually fuzzy sets are overlapped by about 25%.

• It can be observed that in fuzzy logic control the transition from one fuzzy set to another provides a smooth transition from one control action to another.

• If there is no overlapping in the fuzzy sets then the control action would resemble bivalent control (transition from one error region to another is rather abrupt).

• On the other hand if there is too much overlap in the fuzzy sets, there would be a lot of fuzziness and this blurs the distinction in the control action.

For simplicity we can choose the same (triangular) membership functions for each of the fuzzy subsets of all the fuzzy variables.

The example below shows membership functions chosen for 2 fuzzy input variables and 1 output variable in controlling an inverted pendulum.
2.2.3 Knowledge Base

- The knowledge base of a fuzzy logic controller consists of a data base and a rule base.

- The basic function of the data base is to provide the necessary information for the proper functioning of the fuzzification module, the rule base and the defuzzification module.

- This information includes
  - the meaning of the linguistic values of the membership functions of the process state and the control output variables.
  - physical domains and their normalized counterparts together with the normalization, denormalization and scaling factors.
  - the type of the membership function of a fuzzy set.

Fuzzy Control Rules

- The basic function of the rule base is to represented the expert knowledge in a form of if-then rule structure.


- These four methods of deriving the rule base can be described as follows:
  (i) Expert experience and control engineering knowledge
  (ii) Based on operator’s control actions
  (iii) Based on fuzzy model of a process
  (iv) Based on learning
• In this course we will be study how to derive the fuzzy rules based on method (i).
  
(i) **Expert experience and control engineering knowledge**
• This method is the least structured of the four methods and yet it is one of the most widely used today.
• It is based on the derivation of rules from the experience based knowledge of the process operator and/or control engineer.

(ii) **Based on operator’s control actions**
• This method tries to model an operator’s skilled actions or control behavior in terms of fuzzy implications using the input-output data connected with his control actions.
• The idea behind this technique is that it is easier to model an operator’s actions than to model a process, since the input variables of the model are likely found by asking the operator the kind of information he uses in his control actions.

(iii) **Based on fuzzy model of a process**
• In the linguistic approach, the linguistic description of the dynamic characteristics of a controlled process may be viewed as a fuzzy model of the process.
• Based on the fuzzy model, we can generate a set of fuzzy control rules for attaining optimal performance of a dynamic system.
• The set of fuzzy control rules forms the rule base of the fuzzy logic controller.
• Although this approach is somewhat more complicated, it yields better performance and reliability and provides a more tractable structure for dealing theoretically with the fuzzy logic controller.
(iv) Based on learning

- Many fuzzy logic controllers have been built to emulate human decision making behavior.

- Currently, many research efforts are focused on emulating human learning, mainly on the ability to create fuzzy control rules and to modify them based on experience.

- There are now many examples of fuzzy controllers which have the capability to learn and to compose the rules involving neural networks and genetic algorithm.

- One example is given in the research at CAIRO on Self-Organizing Neuro-fuzzy Control Systems (see Part II: Module 4).

Example of rules for traffic control

**IF** traffic from the north of the city is **HEAVY**
**AND** the traffic from the west is **LESS**
**THEN** allow movement of traffic from the north **LONGER**.

**IF** traffic from the north of the city is **AVERAGE**
**AND** the traffic from the west is **AVERAGE**
**THEN** allow **NORMAL** movement of traffic for both sides.
Example of a Rule for a Temperature Control System

If the error in the temperature is positive and large and the rate of change of error is almost zero then heater should be on positive and large.

This rule can be shorten as follows:

IF $e_i$ is PL AND $De$ is ZE THEN $Du$ is PL

or (PL, ZE; PL)

Example 2.1

• For simplification a matrix can be used for presenting the rules which is called fuzzy rule matrix.

• Many control rules are written in the form below:

( Antecedents) (Consequent)

IF $X$ AND $Y$ THEN $Z$

• Example of a fuzzy rule matrix for controlling a temperature control process given as follows
• For the rule $(P, Z; P)$
• For the rule $(N, Z; N)$
• For the rule $(Z, N; N)$

<table>
<thead>
<tr>
<th>Antecedents</th>
<th>N</th>
<th>Z</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>De</td>
<td>N</td>
<td>N</td>
<td>P</td>
</tr>
<tr>
<td></td>
<td>Z</td>
<td>N</td>
<td>Z</td>
</tr>
<tr>
<td></td>
<td>P</td>
<td>N</td>
<td>P</td>
</tr>
</tbody>
</table>

Steady State Rule

Rules?

• How do we formulate the rules??
Derivation of fuzzy rules based on observations

- Though fuzzy rules can be formulated through observation of the process, they are, however, could not be easily derived if one has not much experience.

- A more systematic approach to formulate the fuzzy control rules can be done following the approach given by C.C. Lee.

- The fuzzy rules can be formulated based on the conditions of error, $E$, and the rate of change of error, $\Delta E$, and the output is the change in the control signal, $\Delta u$.

Mathematically, we can write:

for the error at the sampling instant $k$:

- For the derivative of the error in continuous time:
- and in discrete time at sampling instant $k$:
• The prototype of fuzzy control rules is tabulated in Table 1 and a justification of fuzzy control rules is added in Table 2 (These are for only 3 quantizations P, Z and N).

• As an example, based on the underdamped 2nd order system response, the corresponding rule of Region 2 can be formulated as rule $R_1$, below which decreases the system overshoot and for Region 3, Rule $R_2$ has the effect of shortening the rise time.

• More specifically,
  
  $R_1$: if \((E\) is negative and $\Delta E$ is positive) then $\Delta U$ is positive,
  
  $R_2$: if \((E\) is negative and $\Delta E$ is negative) then $\Delta U$ is negative.

• Tables 3 and 4 provide the selection of the consequents for 7 quantizations of the fuzzy control variables (i.e. NB, NM, NS, ZE, PS, PM, PB).

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**Observation of system response for deriving fuzzy control rules**

\[\Delta e(k) = (e(k) - e(k-1))\]
### Table 1: Prototype of Fuzzy Control Rules with Term Sets (NEGATIVE, ZERO, POSITIVE)

<table>
<thead>
<tr>
<th>Rule No.</th>
<th>∆E</th>
<th>∆U</th>
<th>Reference Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P</td>
<td>Z</td>
<td>P a1, a2, a3</td>
</tr>
<tr>
<td>2</td>
<td>Z</td>
<td>N</td>
<td>N d1, d2, d3</td>
</tr>
<tr>
<td>3</td>
<td>N</td>
<td>Z</td>
<td>N c1, c2, c3</td>
</tr>
<tr>
<td>4</td>
<td>Z</td>
<td>P</td>
<td>P b1, b2, b3</td>
</tr>
<tr>
<td>5</td>
<td>Z</td>
<td>Z</td>
<td>Set point</td>
</tr>
</tbody>
</table>

### Table 2: Prototype of Fuzzy Control Rules with Term Sets (NEGATIVE, ZERO, POSITIVE)

<table>
<thead>
<tr>
<th>Rule No.</th>
<th>∆E</th>
<th>∆U</th>
<th>Reference Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>P</td>
<td>N</td>
<td>P 1 (rise time), 5</td>
</tr>
<tr>
<td>7</td>
<td>N</td>
<td>N</td>
<td>N 2 (overshoot), 6</td>
</tr>
<tr>
<td>8</td>
<td>N</td>
<td>P</td>
<td>N 3, 7</td>
</tr>
<tr>
<td>9</td>
<td>P</td>
<td>P</td>
<td>P 4, 8</td>
</tr>
<tr>
<td>10</td>
<td>P</td>
<td>N</td>
<td>P 9</td>
</tr>
<tr>
<td>11</td>
<td>N</td>
<td>P</td>
<td>N 10</td>
</tr>
</tbody>
</table>

### Table 3: Prototype of Fuzzy Control Rules with Term Sets (NB, NM, NS, ZE, PS, PM, PB)

<table>
<thead>
<tr>
<th>Rule No.</th>
<th>∆E</th>
<th>∆U</th>
<th>Reference Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P</td>
<td>B</td>
<td>Z E P B a1</td>
</tr>
<tr>
<td>2</td>
<td>P</td>
<td>M</td>
<td>Z E P M a2</td>
</tr>
<tr>
<td>3</td>
<td>P</td>
<td>S</td>
<td>Z E P S a3</td>
</tr>
<tr>
<td>4</td>
<td>Z</td>
<td>E</td>
<td>N B N B b1</td>
</tr>
<tr>
<td>5</td>
<td>Z</td>
<td>E</td>
<td>N M N M b2</td>
</tr>
<tr>
<td>6</td>
<td>Z</td>
<td>E</td>
<td>N S N S b3</td>
</tr>
<tr>
<td>7</td>
<td>N</td>
<td>B</td>
<td>Z E N B c1</td>
</tr>
<tr>
<td>8</td>
<td>N</td>
<td>M</td>
<td>Z E N M c2</td>
</tr>
<tr>
<td>9</td>
<td>N</td>
<td>S</td>
<td>Z E N S c3</td>
</tr>
<tr>
<td>10</td>
<td>Z</td>
<td>E</td>
<td>ZE PB PB d1</td>
</tr>
<tr>
<td>11</td>
<td>Z</td>
<td>E</td>
<td>ZE PM PM d2</td>
</tr>
<tr>
<td>12</td>
<td>Z</td>
<td>E</td>
<td>ZE PS PS d3</td>
</tr>
<tr>
<td>13</td>
<td>Z</td>
<td>E</td>
<td>ZE ZE ZE set point</td>
</tr>
</tbody>
</table>
2.2.4 The Inference Mechanism

What is an Inference Procedure in FLCS?

[Diagram of the inference mechanism]
Some Notes on the Inference Mechanism

• The Inference Mechanism provides the mechanism for invoking or referring to the rule base such that the appropriate rules are fired.

• There are several methodologies but in Module 1 we have discussed four methods (the compositional operators - Section 1.3.3).

• Two most common methods used in fuzzy logic control are the max-min composition and the max-(algebraic) product composition.

• The inference or firing with this fuzzy relation is performed via the operations between the fuzzified crisp input and the fuzzy relation representing the meaning of the overall set of rules.

• As a result of the composition, one obtains the fuzzy set describing the fuzzy value of the overall control output.

Several Inference Procedures that can be used in FLCS....

- Max-Min
- Max-Algebraic Product (or Max-Dot)
- Max-bounded Product
- Max-Drastic Product
- Max-bounded sum
- Max-algebraic sum
- Max-Max
- Min-Max

2 most commonly used
Example 2.2

• Consider a simple system where each rule comprises two antecedents and one consequent. A fuzzy system with two non-interactive inputs \( x_1 \) and \( x_2 \) (antecedents) and a single output \( y \) (consequent) is described by a collection of \( n \) linguistic if-then propositions:

\[
\text{IF } x_1 \text{ is } A_1^{(k)} \text{ and } x_2 \text{ is } A_2^{(k)} \text{ THEN } y^{(k)} \text{ is } B^{(k)}, \quad k = 1, 2, \ldots, n.
\]

where \( A_1^{(k)} \) and \( A_2^{(k)} \) are fuzzy sets representing the \( k \)th antecedent pairs and \( B^{(k)} \) are the fuzzy sets representing the \( k \)th consequent.

• Based on the Mamdani implication method of inference, and for a set of disjunctive rules, the aggregated output for the \( n \) rules will be given by

\[
\mu_{B^{(k)}(y)} = \text{Max Min}[\mu_{A_1^{(k)}(\text{input}(i))}, \mu_{A_2^{(k)}(\text{input}(j))}]
\]

• The equation above has a simple graphical interpretation, as seen in the accompanying figure.

Example of fuzzy inferencing using Mamdani’s max-min compositional operator

Defuzzification

Rule 1

\[
\begin{align*}
A_1^{(k)} & \quad \text{input}(i) \quad x_1 \\
A_2^{(k)} & \quad \text{input}(j) \quad x_2 \\
B^{(k)} & \quad y
\end{align*}
\]

Rule 2

\[
\begin{align*}
\text{input}(i) \quad x_1 \\
\text{input}(j) \quad x_2 \\

\end{align*}
\]

Defuzzification

\[
\begin{align*}
\text{input}(i) \quad x_1 \\
\text{input}(j) \quad x_2 \\

\end{align*}
\]

\[
\begin{align*}
\text{input}(i) \quad x_1 \\
\text{input}(j) \quad x_2 \\

\end{align*}
\]

\[
\begin{align*}
\text{input}(i) \quad x_1 \\
\text{input}(j) \quad x_2 \\

\end{align*}
\]

\[
\begin{align*}
\text{input}(i) \quad x_1 \\
\text{input}(j) \quad x_2 \\

\end{align*}
\]

\[
\begin{align*}
\text{input}(i) \quad x_1 \\
\text{input}(j) \quad x_2 \\

\end{align*}
\]

\[
\begin{align*}
\text{input}(i) \quad x_1 \\
\text{input}(j) \quad x_2 \\

\end{align*}
\]

\[
\begin{align*}
\text{input}(i) \quad x_1 \\
\text{input}(j) \quad x_2 \\

\end{align*}
\]

\[
\begin{align*}
\text{input}(i) \quad x_1 \\
\text{input}(j) \quad x_2 \\

\end{align*}
\]

\[
\begin{align*}
\text{input}(i) \quad x_1 \\
\text{input}(j) \quad x_2 \\

\end{align*}
\]

\[
\begin{align*}
\text{input}(i) \quad x_1 \\
\text{input}(j) \quad x_2 \\

\end{align*}
\]

\[
\begin{align*}
\text{input}(i) \quad x_1 \\
\text{input}(j) \quad x_2 \\

\end{align*}
\]

\[
\begin{align*}
\text{input}(i) \quad x_1 \\
\text{input}(j) \quad x_2 \\

\end{align*}
\]
Some notes on the max-min inferencing method of Example 2.2

- The figure illustrates the graphical analysis of two rules, where the symbols $A_{11}$ and $A_{12}$ refer to the first and second fuzzy antecedents of the first rule, respectively, and the symbol $B_1$ refers to the fuzzy consequent of the first rule.

- The symbols $A_{21}$ and $A_{22}$ refer to the first and second fuzzy antecedents, respectively, of the second rule, and the symbol $B_2$ refers to the fuzzy consequent of the second rule.

- The minimum function arises because the antecedent pairs given in the general rule structure for this system are connected by a logical "and" connective.

- The minimum membership value for the antecedents propagates through to the consequent and truncates the membership function for the consequent of each rule.

- This graphical inference is done for each rule.

- Then the truncated membership functions for each rule are aggregated.

- The aggregation operation max results in an aggregated membership function comprised of the outer envelope of the individual truncated membership forms from each rule.

- Then, a crisp value for the aggregated output $y^*$ can be obtained through the defuzzification technique which will be discussed later in the next section.
Example 2.3

- In this example we show how another compositional operation results which is based on the Larsen’s Max-product inference technique.
- Based on the max-product implication technique, for a set of disjunctive rules, the aggregated output for the n-th rule would be given by

\[\mu_B^{(k)}(y) = \text{Max} [\mu_{A_1}^{(k)}(\text{input}(i)) \cdot \mu_{A_2}^{(k)}(\text{input}(j))]\]

- The resulting graphical equivalent of the equation above is shown in the following figure.
- The effect of the max-product implication is shown by the consequent membership functions as scaled triangles.
- This figure also shows the aggregated consequent resulting from a disjunctive set of rules and a defuzzified value, \(y^*\), resulting from the defuzzification method.
### Some Notes of Defuzzification

- Defuzzification is a mapping from a space of fuzzy control actions defined over an output universe of discourse into a space of non-fuzzy (crisp) control action.
- This process is necessary because in many practical applications crisp control action is required to actuate the plant.
- The defuzzifier also performs an output denormalization if a normalization is performed in the fuzzification module.
- There are many methods of defuzzification that have been proposed in recent years.
- Unfortunately, there is no systematic procedure for...
Four most common defuzzification methods:

- Max membership method
- Center of gravity method
- Weight average method
- Mean-max membership method

(i) Max membership method
- This scheme is limited to peaked output functions.
- This method is given by the algebraic expression:
  \[ \mu_z(z^*) \geq \mu_z(z) \quad \text{for all } z \in Z \]
(ii) **Center of gravity method**
- This procedure is the most prevalent and physically appealing of all the defuzzification methods.
- It is given by the algebraic expression

\[
\sum \sum = \mu_{z} (z^*)
\]

(iii) **Weight average method**
- This method is only valid for symmetrical output membership functions.
- The weight average method is formed by weighting each membership function in the output by its respective maximum membership value, \( z \).
- It is given by the algebraic expression

\[
\sum = \mu_{z} (z_{1}, z_{2})
\]
(iv) **Mean-max membership method**

- This method is closely related to the first method, except that the locations of the maximum membership can be non-unique.
- This method is given by the expression:

\[ \mu_{a} = \mu_{b} \]

2.3 **Design procedure of the fuzzy logic controller**

- There is a general procedure that can be followed for designing a fuzzy control system.

- Firstly, the designer have to identify the process input and output variables that need to be considered. Thus, one must have a good knowledge on the system to be controlled.

- Next, one should determine on the number of fuzzy partitions (or fuzzy subsets) for the input and output linguistic variables.

- The number of fuzzy partitions of the input-output spaces should be large enough to provide an adequate approximation and yet be small enough to save memory space.

- This number has an essential effect on how fine a control can be obtained.
In the third step, the designer has to choose the membership functions for the input and output fuzzy variables.

There are different types of membership functions available as discussed in the previous module, however, the most common types are triangular, trapezoidal and bell-shaped functions.

After choosing the membership functions, the designer need to derive the fuzzy control rules based on one of the four methods that have been discussed.

In many practical applications method (i) i.e. deriving from expert experience and control engineering knowledge have been used.

Next the inference engine need to be defined, but there is no systematic methodology for realizing the design of an inference engine. Most practitioners use empirical studies and results to provide guidelines for their choices.

Finally one have to choose the right choice of defuzzification method for their particular application.

Finally, once the fuzzy control system has been constructed, the simulation of the system can be carried out.

The performance of the system can be analyzed. If the results are not as desired, changes are made either to the number of the fuzzy partitions or the mapping of the membership functions and then the system can be tested again.

This trial-and-error approach will continue until satisfactory results are obtained.

Although this method is rather time-consuming and nontrivial, it has been widely employed and has been used in many successful industrial applications.
2.4 **Summary of Module 2**

- As fuzzy logic has been successfully applied to many control problems, more than any other areas of applications, in this module we review some control system basics.

- In this module also we discussed some of the reasons behind the success of fuzzy logic control systems.

- The basic components of a fuzzy logic controller has been discussed which include techniques in fuzzification, knowledge base, inference mechanism and defuzzification.

- The design procedure of the fuzzy logic controller is also discussed.

- Although we more specifically discussed fuzzy logic control, the same techniques can be applied for the other areas as well such as pattern recognition and diagnostic systems.