Angle Modulation

Angle modulation encompasses phase modulation (PM) and frequency modulation (FM). The phase angle of a sinusoidal carrier signal is varied according to the modulating signal. In angle modulation, the spectral components of the modulated signal are not related in a simple fashion to the spectrum of the modulating signal. Superposition does not apply and the bandwidth of the modulated signal is usually much greater than the modulating signal bandwidth.

Definitions

A bandpass signal is represented by

\[ S_c(t) = A(t) \cos(\theta(t)) \]  

Where \( A(t) \) is the envelope and \( \theta(t) = \omega_c t + \Phi(t) \). For angle modulation, we can write

\[ S_c(t) = A \cos[2\pi f_c t + \Phi(t)] \]  

Where \( A \) is a constant and \( \Phi(t) \) is a function of the modulating signal. \( \Phi(t) \) is called the instantaneous phase deviation of \( S_c(t) \).

The instantaneous angular frequency of \( S_c(t) \) is defined as

\[ \omega_i(t) = \frac{d\theta(t)}{dt} \]  

In terms of frequency, the instantaneous frequency of \( S_c(t) \) is

\[ f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} \]  

\[ = f_c + \frac{1}{2\pi} \frac{d\Phi(t)}{dt} \]  

\( \frac{1}{2\pi} \frac{d\Phi(t)}{dt} \) is known as the instantaneous frequency deviation. The peak (maximum) frequency deviation is

\[ \Delta f = \max \left| \frac{1}{2\pi} \frac{d\Phi(t)}{dt} \right| = \max |f_i(t) - f_c| \]
Phase Modulation

For PM, the instantaneous phase deviation is proportional to the modulating signal $m_p(t)$:

$$\phi(t) = k_p m_p(t)$$  \hspace{1cm} (7)

where $k_p$ is a constant. Thus, a phase-modulated signal is represented by

$$s_c(t) = A \cos \left[ 2\pi f_c t + k_p m_p(t) \right]$$  \hspace{1cm} (8)

Substituting equation (7) into (5), the instantaneous frequency of $s_c(t)$ can be written as

$$f_i(t) = f_c + \frac{1}{2\pi} k_p \frac{d m_p(t)}{dt}$$  \hspace{1cm} (9)

The peak (maximum) phase deviation is

$$\Delta \phi = \max |\phi(t)| = k_p \max |m_p(t)|$$  \hspace{1cm} (10)

The phase modulation index is given by

$$\beta_p = \Delta \phi$$  \hspace{1cm} (11)

Frequency Modulation

For FM, the instantaneous frequency deviation is proportional to the modulating signal $m_f(t)$:

$$\frac{d \phi(t)}{dt} = k_f m_f(t)$$  \hspace{1cm} (13)

Where $k_f$ is a constant, and

$$\phi(t) = k_f \int_{-\infty}^{t} m_f(\tau) \, d\tau + \phi(-\infty)$$  \hspace{1cm} (14)

$\phi(-\infty)$ is usually set to 0. Thus, a frequency-modulated signal is represented by
\[ S_c(t) = A \cos \left[ 2\pi f_c t + kf \int_{-\infty}^{t} m_f(\tau) \, d\tau \right] \]

Substituting equation (13) into (5), the instantaneous frequency of \( S_c(t) \) can be written as

\[ f_i(t) = f_c + \frac{1}{2\pi} kf m_f(t) \]

Figure (1) shows the modulating signal \( m_f(t) \), the instantaneous frequency \( f_i(t) \), and the FM signal when a saw tooth signal is used as a modulating signal.

**Figure 1:** Frequency modulation: (a) Modulating signal, (b) instantaneous frequency, and (c) FM signal.
The *frequency deviation* from the carrier frequency is

\[ f_d(t) = f_i(t) - f_c = \frac{1}{2\pi} \frac{d\phi(t)}{dt} = \frac{1}{2\pi} k_f m_f(t) \quad 17 \]

And the *peak frequency deviation* is

\[ \Delta f = \max \left| \frac{1}{2\pi} \frac{d\phi(t)}{dt} \right| \]
\[ = \frac{1}{2\pi} k_f \max |m_f(t)| \quad 19 \]

**Generation of Angle-Modulated Signal [2]**

It can be seen from equations (7) and (14) that PM and FM differ only by a possible integration or differentiation of the modulating signal. From equations (7) and (14), we obtain

\[ m_f(t) = \frac{k_p}{k_f} \frac{dm_p(t)}{dt} \quad 20 \]

And

\[ m_p(t) = \frac{k_f}{k_p} \int_{-\infty}^{t} m_f(\tau) d\tau \quad 21 \]

If we differentiate the modulating signal \( m_p(t) \) and frequency-modulate using the differentiated signal, we get a PM signal. On the other hand, if we integrate the modulating signal \( m_i(t) \) and phase-modulate using the integrated signal, we get a FM signal. Therefore, we can generate a PM signal using a frequency modulator or we can generate an FM signal using a PM modulator. This is shown in Figure (2).
Angle Modulation
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Third Class. 2012 – 2013

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Figure 2: Generation of (a) PM using a frequency modulator, and (b) FM using a phase modulator.

Spectrum of an Angle-Modulated Signal

For angle modulation,

\[ s_c(t) = A \cos [2\pi f_c t + \phi(t)] \quad \text{22} \]

And we can write

\[ s_c(t) = \text{Re} \left\{ Ae^{j[2\pi f_c t + \phi(t)]} \right\} = \text{Re} \left\{ Ae^{j2\pi f_c t} e^{j\phi(t)} \right\} \quad \text{23} \]

Expanding \( e^{j\phi(t)} \) in a power series yields

\[ s_c(t) = \text{Re} \left\{ Ae^{j2\pi f_c t} \left[ 1 + j\phi(t) - \frac{\phi^2(t)}{2!} - \ldots + j^n \frac{\phi^n(t)}{n!} + \ldots \right] \right\} \quad \text{24} \]

\[ = A \left[ \cos 2\pi f_c t \phi(t) \sin 2\pi f_c t - \frac{\phi^2(t)}{2!} \cos 2\pi f_c t + \frac{\phi^3(t)}{3!} \sin 2\pi f_c t + \ldots \right] \quad \text{25} \]

It can be seen that the spectrum of an angle-modulated signal consists of an unmodulated carrier plus spectra of \( \phi(t), \phi^2(t), \ldots \), and is not related to the spectrum of the modulating signal in a simple fashion.
Narrowband Angle Modulation

If $\max |\phi(t)| << 1$, we can neglect all higher-power terms of $\phi(t)$ in equation (25) and we have a narrowband angle-modulated signal

$$s_c(t) \approx A[\cos 2\pi f_c t - \phi(t) \sin 2\pi f_c t]$$  \hspace{1cm} (26)

For PM,

$$s_c(t) \approx A[\cos 2\pi f_c t - k_p m_p(t) \sin 2\pi f_c t]$$  \hspace{1cm} (27)

For FM,

$$s_c(t) \approx A\{\cos 2\pi f_c t - [k_f \int_{-\infty}^{t} m_f(\tau) d\tau] \sin 2\pi f_c t\}$$  \hspace{1cm} (28)

Because of the difficulty of analyzing general angle-modulated signals, we shall only consider a sinusoidal modulating signal. Let the modulating signal of a narrowband FM signal be

$$m_f(t) = a_m \cos 2\pi f_m t$$  \hspace{1cm} (29)

Substituting (29) into (14), we have

$$\phi(t) = \frac{k_f a_m}{2\pi f_m} \sin 2\pi f_m t$$  \hspace{1cm} (30)

$$= \beta_f \sin 2\pi f_m t$$  \hspace{1cm} (31)

where $\beta_f = k_f a_m/(2\pi f_m)$. $\beta_f$ is called the frequency modulation index and $\beta_f$ is only defined for a sinusoidal modulating signal. Differentiating (31) and substituting $d\phi(t)/dt$ into (18), we have

$$\beta_f = \frac{\Delta f}{B}$$  \hspace{1cm} (32)

Where $B = f_m$ is the bandwidth of the modulating signal. Substituting equation (31) into (2), we have
\[ s_C(t) = A \cos \left( 2\pi f_c t + \beta f \sin 2\pi f_m t \right) \]

Equation (33) contains the carrier term plus two sideband terms. The bandwidth of the narrowband FM signal is \( 2f_m \) Hz.

For \( \beta f \ll \pi/2 \), \( \cos (\beta f \sin 2\pi f_m t) = 1 \), \( \sin (\beta f \sin 2\pi f_m t) = \beta f \sin 2\pi f_m t \), and

\[ s_C(t) \approx A \left[ \cos 2\pi f_c t - \frac{\beta A}{2} \right] \]

\[ \approx A \cos 2\pi f_c t \quad - \frac{\beta A}{2} \left[ \cos 2\pi (f_c-f_m) t + \cos 2\pi (f_c+f_m) t \right] \]

\[ \approx \text{Re} \left[ e^{j2\pi f_c t} \left( A - \frac{\beta A}{2} e^{-j2\pi f_m t} + \frac{\beta A}{2} e^{j2\pi f_m t} \right) \right] \]

Equation (36) contains the carrier term plus two sideband terms. The bandwidth of the narrowband FM signal is \( 2f_m \) Hz.

In the AM case with sinusoidal modulating signal \( m(t) = a_m \cos 2\pi f_m t \),

\[ s_C(t) = \left[ A + a_m \cos 2\pi f_m t \right] \cos 2\pi f_c t \]

\[ s_C(t) = A \cos 2\pi f_c t + \frac{a_m}{2} \left[ \cos 2\pi (f_c-f_m) t + \cos 2\pi (f_c+f_m) t \right] \]

\[ s_C(t) = A \cos 2\pi f_c t \quad + \frac{mA}{2} \left[ \cos 2\pi (f_c-f_m) t + \cos 2\pi (f_c+f_m) t \right] \]

\[ s_C(t) = \text{Re} \left[ e^{j2\pi f_c t} \left( A + \frac{mA}{2} e^{-j2\pi f_m t} + \frac{mA}{2} e^{j2\pi f_m t} \right) \right] \]

Where the modulation index \( m = a_m/A \). Figure 3 shows the vector representation of a narrowband FM signal and an AM signal.
Figure 3: Vector representation of (a) narrowband FM, and (b) AM.

It can be seen that the resultant of the two sideband vectors in the FM case is always in phase quadrature with the unmodulated carrier, whereas the resultant of the two sideband vectors in the AM case is always in phase with the unmodulated carrier. The distinction and similarity between narrowband FM (or phase modulation) leads us to a commonly used method of generating narrowband angle-modulated signals.
Generation of Narrowband PM and Narrowband FM

The generation of narrowband PM and narrowband FM signals is easily accomplished in view of equations (27) and (28). This is shown in Figure 4.

**Figure 4:** Generation of (a) narrowband PM, and (b) narrowband FM.
Wideband Frequency Modulation (WFM)

Review of Angle Modulation

Angle modulation encompasses phase modulation (PM) and frequency modulation (FM). We have seen that an angle-modulated signal can be represented by.

\[ S_c(t) = A \cos[2\pi f_c t + \phi(t)] \]  

Where \( A \) is a constant. \( \phi(t) \) is a function of the modulating signal and is given by

\[ \phi(t) = \begin{cases} 
  k_p m(t) & \text{for } PM \\
  t & \text{for } FM \\
  k_f \int_{-\infty}^{\infty} m(\lambda) d\lambda & \text{for } FM
\end{cases} \]

Because of the difficulty of analyzing general angle-modulated signals, we shall only consider angle-modulated signals with a sinusoidal modulating signal. Let the modulating signal be

\[ m(t) = \begin{cases} 
  a_m \sin 2\pi f_m t & \text{for } PM \\
  a_m \cos 2\pi f_m t & \text{for } FM
\end{cases} \]

Substituting (3) into (2), we have

\[ \phi(t) = \beta \sin 2\pi f_m t \]

Where

\[ \beta = \begin{cases} 
  k_p a_m & \text{for } PM \\
  k_f a_m & \text{for } FM \\
  \frac{k_f a_m}{2\pi f_m} & \text{for } FM
\end{cases} \]
For FM with a sinusoidal modulating signal, the frequency modulation index is

$$\beta = \frac{\Delta f}{B}$$  \hspace{1cm} (6)

Where $B = f_m$ is the bandwidth of the modulating signal and $\Delta f$ is the peak frequency deviation. The peak frequency deviation is given by

$$\Delta f = \frac{1}{2\pi} = k_f \max |m(t)|$$  \hspace{1cm} (7)

**Wideband Frequency Modulation**

Consider the angle-modulated signal $S_c(t) = A\cos(2\pi f_c t + \beta \sin 2\pi f_m t)$ with sinusoidal modulating signal $m(t) = a_m \cos 2\pi f_m t$. It can be shown that $S_c(t)$ can also be written as

$$S_c(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi (f_c + nf_m) t]$$  \hspace{1cm} (8)

where

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin(x)-nx)} dx$$

The integral is known as the *Bessel function of the first kind of the n-th order* and cannot be evaluated in closed form. Figure 1 shows some Bessel functions for $n = 0, 1, 2, 3, \text{ and } 8$. Clearly, the value of $J_n(\beta)$ becomes small for large values of $n$. 
Figure 1: Bessel functions.

Also, it can be shown from the integral definition of $J_n(\beta)$ that

$$J_n(\beta) = \begin{cases} \frac{1}{\sqrt{\pi \beta}} \left( \frac{\beta^2}{4} \right)^{n/2} \frac{\Gamma(n/2)}{\Gamma(n/2+1/2)} \sin\left(\frac{\pi n}{2} - \beta \right) & n \text{ even} \\ -\frac{1}{\sqrt{\pi \beta}} \left( \frac{\beta^2}{4} \right)^{n/2} \frac{\Gamma(n/2)}{\Gamma(n/2+1/2)} \cos\left(\frac{\pi n}{2} + \beta \right) & n \text{ odd} \end{cases}$$

Therefore, we can write
Figure 2 shows the amplitude spectra of FM signals with a sinusoidal modulating signal and fixed $f_m$.

$$s_c(t) = A \{ J_0(\beta) \cos 2\pi f_c t - J_1(\beta) [\cos 2\pi (f_c - f_m) t - \cos 2\pi (f_c + f_m) t ] + J_2(\beta) [\cos 2\pi (f_c - 2f_m) t + \cos 2\pi (f_c + 2f_m) t ] - J_3(\beta) [\cos 2\pi (f_c - 3f_m) t - \cos 2\pi (f_c + 3f_m) t ] + ... \}$$

See the following observations……..
**Observations:**

1. The spectrum consists of a carrier component at $f_c$ plus sideband components at $f_c + nf_m$ ($n = 1, 2, ...$).
2. The number of sideband terms depends on the modulation index $\beta$.
3. The magnitude of the carrier signal decreases rapidly as $\beta$ increases.
4. The amplitudes of the spectral lines depend on the value of $J_n(\beta)$ (see equation (15)).
5. The bandwidth of the modulated signal with a sinusoidal modulating signal increases as $\beta$ increases, and the bandwidth of the modulated signal is larger than $2\Delta f$.

Figure 3 shows the amplitude spectra of FM signals with a sinusoidal modulating signal and a fixed peak frequency deviation $\Delta f$. Clearly, we get more and more spectral lines crowding into a fixed frequency interval as $f_m$ decreases.

![Figure 3](image-url)

**Figure 3:** Amplitude spectra of FM signals with sinusoidal modulating signal and fixed peak frequency deviation $\Delta f$. 
Bandwidth of Angle-Modulation Signals

From equation (15), we observe that the spectrum consists of a carrier component at \( f_c \) plus an infinite number of sideband components at \( f_c + nf_m \) (\( n = 1, 2, \ldots \)). In fact, 98% of the normalized total signal power is contained in the bandwidth

\[
B_T = 2(\beta + 1)B
\]

Where \( \beta \) is either the phase modulation index or the frequency modulation index and \( B \) is the bandwidth of the modulating signal. The bandwidth of the angle-modulated signal with sinusoidal modulating signal depends on \( \beta \) and \( B \). This is called Carson’s rule. It gives a rule-of-thumb expression and an easy way to evaluate the transmission bandwidth of angle-modulated signals. When \( \beta \ll 1 \), the signal is a narrowband angle-modulated signal and its bandwidth is approximately equal to \( 2B \).

Generation of Wideband FM

*Indirect Method.*

In this method, a narrowband frequency-modulated signal is first generated using an integrator and a phase modulator. A frequency multiplier is then used to increase the peak frequency deviation from \( \Delta f \) to \( n\Delta f \). Use of frequency multiplication normally increases the carrier frequency from \( f_c \) to \( nf_c \). A mixer or double-sideband modulator is required to shift the spectrum down to the desired range for further frequency multiplication or transmission. This is shown in Figure 4.
**Direct Method.**

Here the carrier frequency is directly varied in accordance with the modulating signal. A common method used for generating direct FM is to vary the inductance $L$ or capacitance $C$ of a *voltage-controlled oscillator (VCO)*. This is shown in Figure 5.

\[
C = k m(t) + C_0 \\
C = \Delta C + C_0
\]

Where

\[\Delta C = k m(t)\]

**Figure 4:** Indirect method of generating WFM.

**Figure 5:** Direct method of generating WFM.
$k$ is a constant and $C_0$ is the capacitance of the VCO when the input signal to the oscillator is zero. The instantaneous frequency is given by

\[
f_i = \frac{1}{2\pi \sqrt{LC}}
\]

\[
f_i = \frac{1}{2\pi \sqrt{LC}} \sqrt{1 + \frac{\Delta C}{C_0}}
\]

\[
f_i = f_c \left(1 + \frac{\Delta C}{2C_0}\right)^{-1/2}
\]

Where the zero-input-signal resonance frequency is

\[
f_c = \frac{1}{2\pi \sqrt{LC}}
\]

For $\Delta C \ll C_0$, we can write

\[
f_i \approx f_c \left(1 - \frac{\Delta C}{2C_0}\right)
\]

\[
f_i \approx f_c \left(1 - \frac{km(t)}{2C_0}\right)
\]

\[
f_i \approx f_c - \Delta f
\]

Where

\[
\Delta f = \frac{km(t)}{2C_0} f_c = \frac{\Delta C}{2C_0} f_c
\]

Although the change in capacitance may be small, the frequency deviation $\Delta f$ may be quite large if the resonance frequency $f_c$ is large. We can alternatively vary the inductance to achieve the same effect.
**Advantage** - Large frequency deviations are possible and thus less frequency multiplication is needed.

**Disadvantage** - The carrier frequency tends to drift and additional circuitry is required for frequency stabilization.

To stabilize the carrier frequency, a **Phase-Locked Loop** can be used. This is shown in Figure 6.

Figure 6: Direct method of generating WFM with frequency stabilization.
Angle Demodulation

Review of Angle Modulation

Angle modulation encompasses phase modulation (PM) and frequency modulation (FM). We have seen that an angle-modulated signal can be represented by

\[ S_c(t) = A \cos \theta(t) \]  
\[ S_c(t) = A \cos[2\pi f_c t + \phi(t)] \]

where \( A \) is a constant, \( \theta(t) = 2\pi f_c t + \phi(t) \), and \( f_c \) is the carrier frequency. \( \phi(t) \) is a function of the modulating signal \( m(t) \) and is given by

\[
\phi(t) = \begin{cases} 
    k_p m(t) & \text{for PM} \\
    t & \text{for FM} \\
    k_f \int_{-\infty}^{t} m(\lambda) d\lambda & \text{for FM}
\end{cases}
\]

And

\[
\frac{d\phi(t)}{dt} = \begin{cases} 
    k_p \frac{dm(t)}{dt} & \text{for PM} \\
    k_f m(t) & \text{for FM}
\end{cases}
\]

The instantaneous angular frequency of \( S_c(t) \) is defined as

\[ \omega_i(t) = \frac{d\theta(t)}{dt} \]

In terms of frequency, the instantaneous frequency of \( S_c(t) \) is

\[ f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} = f_c + \frac{1}{2\pi} \frac{d\phi(t)}{dt} \]
Angle Demodulation

Demodulation of an angle-modulated signal requires a circuit that produces an output proportional to the instantaneous frequency \( f_i(t) \) or the instantaneous frequency deviation \( \frac{1}{2\pi} \frac{d\phi(t)}{dt} \) of the input signal to the demodulator.

**Frequency Discrimination.**

Frequency discrimination is a frequency-to-amplitude conversion process. Consider an angle-modulated signal

\[
S_c(t) = A \cos \theta(t)
\]

Where

\[
\theta(t) = 2\pi f_c t + \phi(t)
\]

\[
\phi(t) = \begin{cases} 
  k_p m(t) & \text{for PM} \\
  k_f \int_{-\infty}^{t} \frac{m(\lambda)}{d\lambda} d\lambda & \text{for FM}
\end{cases}
\]

And

\[
\frac{d\phi(t)}{dt} = \begin{cases} 
  k_p \frac{dm(t)}{dt} & \text{for PM} \\
  k_f m(t) & \text{for FM}
\end{cases}
\]
If we differentiate equation (9), we get

\[
s'_c(t) = -A \sin \theta(t) \frac{d\theta(t)}{dt}
\]

\[
= -A \sin \theta(t) \left[2\pi f_c + \frac{d\phi(t)}{dt}\right]
\]

\[
=-A \left[2\pi f_c + \frac{d\phi(t)}{dt}\right] \sin \left[2\pi f_c t + \phi(t)\right]
\]

The signal is both amplitude- and angle-modulated. If we pass the signal to an envelope detector, we get

\[
y(t) = A \left[2\pi f_c + \frac{d\phi(t)}{dt}\right]
\]

\[
y(t) = \begin{cases} 
A \left[2\pi f_c + k_p \frac{dm(t)}{dt}\right] & \text{for PM} \\
A \left[2\pi f_c + k_f m(t)\right] & \text{for FM}
\end{cases}
\]

Knowing the values of \(A, f_c, k_p,\) and \(k_f,\) we can compute the desired signal \(m(t)\) from \(y(t).\) Figure 1 shows the circuit for frequency demodulation. The differentiator followed by an envelope detector is called a \textit{frequency discriminator}. For demodulation of PM signals, we simply integrate the output of a frequency discriminator. This yields a signal which is proportional to \(m(t).\) Figure 2 shows the circuit for phase demodulation.
In practice, channel noise and other factors may cause $A$ to vary. If $A$ varies, $y(t)$ will vary with $A$. Hence, it is essential to maintain the amplitude of the input signal to the frequency discriminator. A *hard limiter* is usually used to eliminate any amplitude variations. A hard limiter is a device which limits the output signal to (say) +1 or -1 volt. Figure 3 shows the input-output characteristic of a hard limiter.
**Zero-Crossing Detection.**

We have seen that a hard limiter is usually used to eliminate any amplitude fluctuation. The message signal must therefore be contained in the points where the angle-modulated signal crosses the zero voltage level. This produces a means of demodulating an angle-modulated signal. Consider the angle-modulated signal as shown in Figure 4.

Let \( t_1 \) and \( t_2 \) be two adjacent zero-crossing points, where \( t_2 > t_1 \). Integrating equation (6), we have

\[
\int_{t_1}^{t_2} d\theta(t) = \int_{t_1}^{t_2} 2\pi f_i(t) dt
\]
\[
\theta(t_2) - \theta(t_1) = \begin{cases} 
\int_{t_1}^{t_2} \left[ 2\pi f_c + k \frac{dm(t)}{dt} \right] dt & \text{for PM} \\
\int_{t_1}^{t_2} \left[ 2\pi f_c + k f_m(t) \right] dt & \text{for FM}
\end{cases}
\]

but also
\[
\theta(t_2) - \theta(t_1) = \pi
\]

For \( f_c \gg B \) (the bandwidth of the message signal), \( \frac{dm(t)}{dt} \) for PM signals and \( m(t) \) for FM signals change much more slowly than \( f_c \). \( \frac{dm(t)}{dt} \) and \( m(t) \) may be assumed constant in the interval \( t_2 - t_1 \). We can write
\[
\pi \approx \begin{cases} 
\left[ 2\pi f_c + \frac{k}{p} \frac{dm(t)}{dt} \right] (t_2 - t_1) & \text{for PM} \\
\left[ 2\pi f_c + k f_m(t) \right] (t_2 - t_1) & \text{for FM}
\end{cases}
\]

\[
\pi = 2\pi f_i(t) \left[ t_2 - t_1 \right]
\]

\[
f_i(t) = \frac{1}{2} \left( \frac{t_2}{t_1} \right)
\]

Where
\[
f_i(t) = \begin{cases} 
f_c + \frac{1}{2\pi} k \frac{dm(t)}{dt} & \text{for PM} \\
f_c + \frac{1}{2\pi} k f_m(t) & \text{for FM}
\end{cases}
\]
Knowing the values of $f_c, k_p$ and $k_f$, the desired signal $m(t)$ may be found by measuring the spacing between zero crossings in the interval $t_2 - t_1$. A detector utilizing this technique is called a *zero-crossing detector*. For demodulation of PM signals, we simply integrate the output of a zero-crossing detector. Again, this yields a signal which is proportional to $m(t)$.

In practice, we consider counting $n$ number of zero-crossings in a time interval $T$, where

$$\frac{1}{f_c} \ll T \ll \frac{1}{B}$$

and $B$ is the bandwidth of the message signal. This is shown in Figure 5.

![Figure 5: Counting intervals.](image)

Then, the number of zero crossings in a time interval $T$ is

$$n = \frac{T}{T_2 - T_1}$$

and

$$\frac{n}{T} = f_i(t)$$