Introduction: The term *microwaves* refers to alternating current signals with frequencies between 300 MHz and 300 GHz, with corresponding electrical wavelength between 1m and 1mm, $\lambda = c/f$. Figure (1) shows the location of the microwave frequency band in the electromagnetic spectrum.

<table>
<thead>
<tr>
<th>Typical Frequencies</th>
<th>Approximate Band Designations</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM broadcast band</td>
<td>535-1605 KHz</td>
</tr>
<tr>
<td>Short wave radio band</td>
<td>3-30 MHz</td>
</tr>
<tr>
<td>FM broadcast band</td>
<td>88-108 MHz</td>
</tr>
<tr>
<td>VHF TV (2-4)</td>
<td>54-72 MHz</td>
</tr>
<tr>
<td>VHF TV (5-6)</td>
<td>76-88 MHz</td>
</tr>
<tr>
<td>UHF TV (7-13)</td>
<td>174-216 MHz</td>
</tr>
<tr>
<td>UHF TV (14-83)</td>
<td>470-890 MHz</td>
</tr>
<tr>
<td>US cellular telephone</td>
<td>824-849 MHz</td>
</tr>
<tr>
<td>European GSM cellular</td>
<td>880-915 MHz</td>
</tr>
<tr>
<td>GPS</td>
<td>1575.42 MHz</td>
</tr>
<tr>
<td>Microwave ovens</td>
<td>2.45 GHz</td>
</tr>
</tbody>
</table>

Figure (1) The electromagnetic spectrum
Microwave Applications

1. Medical: Imaging, selective heating, sterilization etc.
2. Domestic/industrial: Cooking, traffic & toll management, sensor
3. Surveillance: Electronic warfare, security system etc.
4. Radar: Air defense, guided weapon, collision avoidance, weather
5. Astronomy & Space exploration: Monitor and collect data.
6. Communication: Satellite, Space, Long distance telephone, etc.

Electromagnetic Waves

During the early stages of studies of electric and magnetic phenomena, they were thought to be unrelated. In 1865, James Clark Maxwell provided a mathematical theory that showed a close relation between electric and magnetic phenomena. His theory is based upon the following 4 pieces of information:

1. Electric fields originate on positive charges and terminate on negative charges.
2. Magnetic field lines always form closed loops.
3. A varying magnetic field induces an emf and hence an electric field (Faraday’s law of induction).
4. Magnetic fields are generated by moving charges.

Properties of Electromagnetic Waves

1. The electric and magnetic fields in an electromagnetic wave are in phase.
2. The electric and magnetic fields are perpendicular to each other.
3. The electric and magnetic fields are in planes perpendicular to the direction of travel of the wave. They are transverse waves (TEM).
4. Electromagnetic waves travel at the speed of light:

\[ c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \]
where $\mu_0 = 4\pi \times 10^{-7}$ N $\cdot$ m/A and $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m is the permittivity of free space. Numerically, $c = 2.99792 \times 10^8$ m/s.

**Q/** What is the mean of TE-mode wave?

**A/** Transverse electric (TE) modes: no electric field in the direction of propagation. These are sometimes called H modes because there is only a magnetic field along the direction of propagation (H is the conventional symbol for magnetic field).

**Maxwell Equations**

The result of combining Faraday's Law, Amper's Law and Gauss Laws are referred to Maxwell's equations:

<table>
<thead>
<tr>
<th>Integral form</th>
<th>Differential form</th>
</tr>
</thead>
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<tr>
<td>$\oint_c \vec{E} \cdot d\vec{L} = \frac{-d}{dt} \oint_s \vec{B} \cdot ds$</td>
<td>$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (Faraday’s Law)</td>
</tr>
<tr>
<td>$\oint_c \vec{H} \cdot d\vec{L} = \oint_s \vec{j} \cdot ds + \frac{d}{dt} \oint_s \vec{D} \cdot ds$</td>
<td>$\nabla \times \vec{H} = \vec{j} + \frac{\partial \rho}{\partial t}$ (Amper's Law)</td>
</tr>
<tr>
<td>$\oint_s \vec{D} \cdot d\vec{s} = \int_v \rho_v \cdot dv$</td>
<td>$\nabla \cdot \vec{D} = \rho_v$ (Gauss Law for electric field)</td>
</tr>
<tr>
<td>$\oint_s \vec{B} \cdot d\vec{s} = 0$</td>
<td>$\nabla \cdot \vec{B} = 0$ (Gauss Law for magnetic field)</td>
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**1- Faraday’s Law**

The induction of electromagnetic force (*emf*) by changing magnetic flux was first observed by Faraday and Henry. The results of a large number of experiments can be summarized by associating an *emf*.

$$emf = \frac{-d\Phi}{dt} \quad \text{..........................(1)}$$

With the time varying in magnetic flux through a circuit.

where $\Phi$ is the magnetic flux

$t$ is the time
Eq. (1) represent the Faraday's Law of electromagnetic induction. Which is found to be independent of the way in which the flux is changed.

\[\text{emf} = \oint \vec{E} \cdot d\vec{L} \] ................(2)

\[\psi = \oint \vec{B} \cdot ds \] ..................(3)

Sub Eq. (2,3) in Eq. (1)

\[\oint \vec{E} \cdot d\vec{L} = -\frac{d}{dt} \int_s \vec{B} \cdot ds \] ..................(4)

If the circuit is a rigid stationary circuit, the time derivative can be taken inside the integral where it become a partial derivative.

\[\oint \vec{E} \cdot d\vec{L} = -\int \frac{\partial \vec{B}}{\partial t} \cdot ds \] .........................(5)

From stokes theorem which can be used to transform the line integral of \( E \) in to the surface integral curl \( E \).

\[\oint \vec{E} \cdot d\vec{L} = \int_s \nabla \times \vec{E} \cdot ds \] ..................(6)

\[\int_s (\nabla \times \vec{E}) \cdot ds = \int_s \left(\frac{\partial \vec{B}}{\partial t}\right) \cdot ds \] ..................(7)

Which is the differential form of Faraday's Law. The – ve sign in Faraday's Law indicates, as can be easily demonstrated that the direction of induced \( \text{emf} \) is such as to oppose the change that produce it.

2- Amper's Law

\[\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \]

Which gives the modified Amper's Law

The introduction of the second term on the right which is known displacement current represent one of Maxwell's major contributions to electromagnetic fields.
Example:

Compare the conducting and displacement current densities in copper ($\varepsilon = \varepsilon_0$, $\mu = \mu_0$, and $\sigma = 5.8 \times 10^7$ s/m at 1 MHz) assuming sinusoidal variation of electric field in the material $E = E_0 \sin \omega t$ v/m.

Solution

\[ J_c = \sigma E_0 \sin \omega t \]

\[ J_d = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} = \varepsilon E_0 \omega \cos \omega t \]

\[ \frac{J_{c\text{ max}}}{J_{d\text{ max}}} = \frac{\sigma E_0}{\varepsilon E_0 \omega} = \frac{\sigma}{\varepsilon \omega} \]

\[ \frac{J_{c\text{ max}}}{J_{d\text{ max}}} \text{copper} = \frac{\sigma}{\varepsilon \omega} = \frac{\sigma}{2\pi f \varepsilon} = \frac{5.8 \times 10^7}{2\pi \times 10^6 \times 8.85 \times 10^{-12}} \approx 10^{12} \]

\[ \frac{J_{c\text{ max}}}{J_{d\text{ max}}} \text{Teflon} = \frac{\sigma}{\varepsilon \omega} = \frac{\sigma}{2\pi f \varepsilon} = \frac{3 \times 10^{-8}}{2\pi \times 10^6 \times 2.1 \times 8.85 \times 10^{-12}} \approx 2.57 \times 10^{-9} \]

H.W.: From Maxwell's Equations, derive the $c=3\times10^8$ m/s

1-4 Poynting Vector and Flow of Power

As electromagnetic waves propagate through space from their source to distance receiving points, there is a transfer of energy from the source to the receivers. There exists a simple and direct relation between the rate of this energy transfer and the amplitudes of the electric and magnetic field strengths of the electromagnetic waves. This relation can be obtained from Maxwell’s equation as follows.

\[ J = \nabla \times H - \frac{\partial D}{\partial t} = \nabla \times H - \varepsilon \dot{E} \]

...............(1)

Multiply the above equation by dot E

\[ E \cdot J = E \cdot \nabla \times H - \varepsilon E \cdot \dot{E} \]

...............(2)

By using identity

\[ E \cdot \nabla \times H = H \cdot \nabla \times E - E \cdot \nabla \times H \]

...............(3)

\[ E \cdot J = H \cdot \nabla \times E - E \cdot \nabla \times H - \varepsilon E \cdot \dot{E} \]

...............(4)
From the second field equation
\[ \nabla \times E = -\frac{\partial B}{\partial t} = -\mu \dot{H} \] ...............(5)

Put equation (5) in equation (4) yield
\[ E.J = -\mu H \dot{H} - \varepsilon E \dot{E} - \nabla \times E \times H \] ...............(6)

Now using identity
\[ H \dot{H} = \frac{1}{2} \frac{\partial}{\partial t} H^2 \quad E \dot{E} = \frac{1}{2} \frac{\partial}{\partial t} E^2 \] ...............(7)

Put equations (7) in equation (6) yield
\[ E.J = -\mu \frac{1}{2} \frac{\partial}{\partial t} H^2 - \varepsilon \frac{1}{2} \frac{\partial}{\partial t} E^2 - \nabla \times E \times H \] ...............(8)

Integrating over a volume \( V \)
\[ \int_V E.J \ dv = -\frac{\partial}{\partial t} \int_V \left( \mu \frac{1}{2} H^2 + \varepsilon \frac{1}{2} E^2 \right) \ dv - \int_V \nabla \times E \times H \ dv \] ...............(9)

Using divergence theorem, the last term can be change from a volume integral to a surface integral over the surface \( S \) surrounding \( V \), that is
\[ \int_V \nabla \times H \ dv = \oint_S E \times H \cdot ds \] ...............(10)

Put equation (10) in equation (9) yield
\[ \int_V E.J \ dv = -\frac{\partial}{\partial t} \int_V \left( \mu \frac{1}{2} H^2 + \varepsilon \frac{1}{2} E^2 \right) \ dv - \oint_S E \times H \cdot ds \] ...............(11)

\[ \int_V E.J \ dv \] this equation is represents instantaneous power dissipated in volume \( V \).
\[ \mu \frac{1}{2} H^2 \] this equation is represents stored magnetic energy per unit volume.
\[ \varepsilon \frac{1}{2} E^2 \] this equation is represents stored electric energy per unit volume.

The interpretation of the remaining term follows from the application of the law of conservation of energy. The rate of energy dissipation in the volume \( V \) must equal the rate at which the stored energy in \( V \) is decreasing, plus the rate at which energy is entering the volume \( V \) from outside. The term
\[ -\oint_S E \times H \cdot ds \]
Therefore must represent the rate of flow of energy inward through the surface of the volume. Then this expression without the negative sign, represents rate of flow of energy outward through the surface enclosing the volume.

The interpretation of the equation (10) leads to the conclusion that the integral of $E \times H$ over any closed surface gives the rate of energy flow through that surface. It is seen that the vector

$$S = E \times H \text{ watt/ square meter}$$

The direction of flow $S$ is perpendicular to $E$ and $H$

**Example:**

Find power flow in a concentric cable. Consider the transfer of power to a load resistance $R$ along a concentric cable which has a d-c voltage $V$ between conductors and a steady current $I$ flowing in the inner and outer conductors. The radius of the conductor is $a$ and the (inside) radius of the outer conductor is $b$. 
**Solution:**

The magnetic field strength $H$ will be directed in the circles about the axis. By Amper's Law the magneto motive force around any of these circles will be the current enclosed, that is

$$H = \frac{1}{2\pi r}$$

The electric field strength $E$ will be directed radially

$$V = \frac{q}{2\pi \varepsilon} \log \frac{b}{a} \text{ volt}$$

and $E$ is given

$$E = \frac{q}{2\pi \varepsilon r} \text{ volt/meter}$$

Therefore $E$ is

$$E = \frac{V}{r \log \frac{b}{a}}$$

The poynting Vector is

$$S = E \times H \quad \text{w/m}^2$$

The total power flow along the cable will be given by the integration of the poynting Vector over any cross-section surface

$$P = \int S \frac{V}{r \log \frac{b}{a}} \left( \frac{1}{2\pi r} \right) 2\pi r \, dr = \frac{VI}{\log \frac{b}{a}} \int_a^b \frac{dr}{r} = VI \quad \text{watt}$$

**Example:**

Find power flow in a conductor having resistance carries a direct current $I$, there will be a value of $E$ within the conductor. It will be parallel to the direction of the current ($E = \frac{I}{\sigma}$), so there will still be no radial component of $P$. Consider a wire of length $L$ having a voltage drop $V_L$ along the wire.

**Solution:**

The wire will be parallel to the $z$-axis. Then $E$ in the wire and at its surface

$$E_z = \frac{V_L}{L}$$
The magnetic field strength $H$ will be in $\emptyset$ direction and at the surface of the wire it will have a value

$$H = \frac{1}{2\pi a}$$

Where $a$ is the radius of the wire. $E_z$ and $H_\emptyset$ are at right angles, so the poynting vector will have a magnitude

$$S = E_z H_\emptyset$$

and the total power flowing into the wire through the surface will be

$$P = \int S \cdot ds = \int_0^L E_z H_\emptyset \ 2\pi a \ dz = \frac{V_L}{L} \int_0^L dz = V_L I \ \text{watt}$$

1-5 Instantaneous, Average, and Complex poynting Vector

$$P_{\text{inst}} = E \times H$$

$$E = \Re (E e^{j\omega t})$$

$$H = \Re (H e^{j\omega t})$$

$$E = \frac{1}{2} \left[ E e^{j\omega t} + E^* e^{-j\omega t} \right]$$

$$H = \frac{1}{2} \left[ H e^{j\omega t} + H^* e^{-j\omega t} \right]$$

$$P_{\text{inst}} = \frac{1}{4} \left[ E \times H^* + E^* \times H \right] + \frac{1}{4} \left[ E \times H e^{j2\omega t} + E^* \times H^* e^{-j2\omega t} \right]$$

Let

$$M = E \times H^*$$

$$M^* = (E \times H^*)^* = E^* \times H$$
\[ N = E \times H \]
\[ N^* = (E \times H)^* = E^* \times H^* \]

Then
\[ P_{\text{inst}} = \frac{1}{4} [M + M^*] + \frac{1}{4} [N_{\text{e}j^{2wt}} + N^*_{e^{-j^{2wt}}}] \] ..................(4)
\[ P_{\text{inst}} = \frac{1}{2} \text{Re} \ (M) + \frac{1}{2} \text{Re} \ (N_{e^{j^{2wt}}}) \]
\[ P_{\text{ave}} = \frac{1}{T} \int_{0}^{T} P_{\text{inst}} \ dt \] ..................(5)
\[ P_{\text{ave}} = \frac{1}{T} \int_{0}^{T} \frac{1}{2} \text{Re} \ (E \times H^*) dt + \frac{1}{T} \int_{0}^{T} \frac{1}{2} \text{Re} \ (E \times H) e^{j^{2wt}} dt \]
\[ P_{\text{ave}} = \frac{1}{2} \text{Re} \ (E \times H^*) + \frac{1}{2T} \text{Re} \ (E \times H) \int_{0}^{T} \cos(2wt) \ dt \]
\[ P_{\text{ave}} = \frac{1}{2} \text{Re} \ (E \times H^*) \]
\[ P_{\text{ave}} = \frac{1}{2} \text{Re} \ (S) \]

Where
\[ S = \frac{1}{2} E \times H^* \] ..................(6)

\( S \) is the complex poynting vector

**Example:**

Suppose that the field vectors of a wave in free space are given by:

\[ E = 100 \ \cos \left( wt + \frac{4\pi}{3} \ x \right) \ \text{az} \]
\[ H = \frac{100}{120\pi} \ \cos \left( wt + \frac{4\pi}{3} \ x \right) \ \text{ay} \]

\( f = 200 \ \text{MHz} \)

determine the direction of power flow and the average power crossing the surface area bounded by
\[ 0 \leq y \leq 2 \]
\[ 0 \leq z \leq 2 \]
Solution:

\[ S = \frac{1}{2} E \times H^* \]

\[ = \frac{1}{2} \left[ \frac{100^2}{120\pi} \cos^2 \left( wt + \frac{4\pi}{3} x \right) \right] a_x \]

The direction of power flow is

\[ P_{ave} = \frac{1}{2} \text{Re} (S) = 13.262 \cos^2 \left( wt + \frac{4\pi}{3} x \right) a_x \]

\[ P_{ave} = \int S \, ds = 13.262 \cos^2 \left( wt + \frac{4\pi}{3} x \right) \int_0^2 dy \int_0^2 dz \]

\[ = 53.262 \cos^2 \left( wt + \frac{4\pi}{3} x \right) \]