In order to transmit signals, such as speech and video signals, we have to digitize them first. The process of converting analog to digital is called analog-to-digital conversion (ADC). ADC involves three procedures, sampling, quantizing, and binary encoding. In this lecture, the concept of sampling and sampling theorem will be given first and then all three ADC conversion procedures will be discussed under pulse code modulation (PCM).

**Sampling**

The main job of sampling is to convert continuous time domain signal into a discrete signal. This section discusses the process of sampling.

Suppose we want to sample the signal shown in figure 1 (a) named as $m(t)$. The first thing to specify is the sampling interval $T_s$, then the sampled signal will have infinite sequence of samples $m(nT_s)$ where $n=0,1,2,…$(Integer number). The reciprocal of the sampling interval $T_s$ is called sampling rate and can be calculated using the formula:

$$\text{Sampling rate } (f_s) = \frac{1}{T_s} \text{ Samples per second} \quad (1)$$

Using the unit impulse train $\delta_T(t)$ shown in figure 1 (b), the sampled signal can be expressed as:

$$m_s(t) = m(t)\delta_T(t) = m(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$= \sum_{n=-\infty}^{\infty} m(t)\delta(t - nT_s) = \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t - nT_s) \quad (2)$$

**Hint:** In the previous equation, the property of impulse function is used to generate the sampled signal and that property is:

$$m(t)\delta(t - t_0) = m(t_0)\delta(t - t_0) \quad (3)$$
Figure 1 shows the original, unit impulse train, and the sampled signal.

![Figure 1](image.png)

Figure 1: (a), (b), and (c) show original signal, impulse train, and sampled signal respectively.

**Sampling theorem:**

The sampling theorem states that the message signal \( m(t) \) “can be uniquely determined from its value \( m(nT_s) \), sampled at uniform interval \( T_s \) if \( T_s \leq \frac{\pi}{w_m} \) = \( 1/(2f_m) \).”

\[
m(t) = \sum_{n=-\infty}^{\infty} m(nT_s) \frac{\sin(\omega_M(t - nT_s))}{\omega_M(t - nT_s)} \tag{4}
\]

- In literature, \( m(t) \) is called Nyquist-Shannon interpolation formula or sometimes called cardinal series.
- \( T_s \) is well known as Nyquist interval = \( 1/(2f_m) \).
- \( f_s \) is the sampling rate (or Nyquist rate) and its minimum value is \( 1/(T_s) = 2f_m \).
**Important definitions:**

- **Band-limited signals:** Signals whose Fourier transform $M(w)$ satisfies the following condition,

\[
M(w) = 0 \quad \text{for } |M| > w_m \tag{5}
\]

Where, $w_m = 2\pi f_m$

Band limited signals is also named as low-pass signal. An example of band limited signal is shown in figure 2.

![Figure 2](image)

Figure 2 shows a band-limited signal

- **Band pass signals:** Signals whose Fourier transform $M(w)$ satisfies the condition,

\[
M(w) = 0 \quad \text{except for } \begin{cases} 
  w_1 < w < w_2 \\
  -w_2 < w < -w_1 
\end{cases} \tag{6}
\]

Where, $w_1 = 2\pi f_1$ and $w_2 = 2\pi f_2$

An example of band limited signal is shown in figure 3.

![Figure 3](image)

Figure 3 shows a band-pass signal
**Pulse Code Modulation (PCM)**

It is an analog to digital conversion method by which the information contained in the instantaneous samples of the analog signals is represented by digital words in a serial bit stream. Assuming that the number of binary digits is $n$, then the number of possible unique code words is $M = 2^n$. For example, if we have three binary digits, we would have 8 possible code words to represent all amplitude levels.

PCM is so popular because of the unique aspects it provides. Some of the advantages of PCM are listed below:

- Relatively inexpensive digital circuits can be used to build the system.
- Time division multiplexing (TDM) can be used to time division multiplex the digital signals with PCM signals (originally are audio or video signals).
- When transmitting for long distances, like long-distance digital telephone systems, repeaters are required to regenerate the signal. In case of PCM signal, the repeater can regenerate the noisy PCM to produce a clean PCM signal; however, we may still get errors because of noise.
- The noise performance of digital systems is better than the noise performance of analog systems. Moreover, errors and noise effects in digital systems can be reduced even more using proper coding techniques.

**Question: What is the only disadvantage of PCM?**

**Generation of PCM:** The generation of PCM signal involves three main steps which are clearly shown in figure 4.

![Diagram of PCM signal generation](image)

Figure 4 shows the generation of PCM signal
1 Sampling

Three different sampling methods can be implemented as it is shown in figure 5.

![Sampling Methods](image)

Figure 5 shows the three possible sampling methods can be used in PCM.

**NOTE:** We should keep in mind that the sampling rate must be at least 2 times compared to the highest frequency contained in the signal (Nyquist theorem).

**Question:** What will happen when we do sampling with a rate of:

1. $f_s = 2f_m$
2. $f_s = 4f_m$
3. $f_s = 0.5f_m$

**Solution:** As we see in figure (6):

- Sampling at a rate of $f_s = 2f_m$ generates good estimate of the original signal.
- Sampling at a rate of $f_s = 4f_m$ generates good estimate of the original signal, however, with redundant unnecessary samples (over sampling).
- Sampling at a rate of $f_s = 0.5f_m$ generates a sampled signal which does not look like the same as the original signal was (under sampling).
Figure 6 shows how sampling rate affects on the sampled signal

**Question:** Based on Nyquist theorem, what is the sampling rate the telephone companies should use to sample and digitize voice? \( f_m = 4000 \) Hz

2 Quantization

The sampling process results in a series of samples with varying amplitude between minimum and maximum amplitude values. The next step is quantization which gives a finite set of known amplitude values to the infinite amplitude values produced by the sampler. There are two different types of quantization, uniform and non uniform.

- **Uniform quantization:** The quantizer divides the distance between the max and min amplitudes into equal \( L \) zones and each one of these zones has a height of \( \Delta \) where,

\[
\Delta = \frac{\text{max-min}}{L} \quad (7)
\]

**Example:** \( V_{\text{max}} = 20 \text{ v} \), \( V_{\text{min}} = -20 \text{ v} \) and we want to use 8 quantization levels \((L=8)\), then

\[
\Delta = \frac{20-(-20)}{8} = 5
\]

which is the step size and the levels will be:

\((-20,-15), (-15,-10),...,(15,10).\)
Two well known types of uniform quantization are shown in figure 7

![Figure 7: Two types of uniform quantization](image)

**Example:** Quantize the following sequence \{1.2, -0.2, -0.5, 0.4, 0.89, 1.3\} using uniform quantizer whose a range of \((-1.5,1.5)\) with four levels.

**Solution:**

\[
\Delta = \frac{\text{max} - \text{min}}{L} = \frac{1.5 - (-1.5)}{4} = 0.75
\]

then,

- 1.2 falls between 0.75 and 1.5 .............. quantized value = 1.125
- -0.2 falls between 0 and –0.75 .............. quantized value = -0.375
- -0.5 falls between 0 and –0.75 .............. quantized value = -0.375
- And so on till you get the following quantized sequence,

\{1.125, -0.375, -0.375, 1.125, 1.125\}
Non uniform quantization: Researchers have proved that:

- "Real audio signals (speech and music) are more concentrated near zeros."
- "Human ear is more sensitive to the quantization errors at small values"

Therefore, to handle these problems, researchers suggest non-uniform quantization where the quantization step size is smaller near zero and it gets larger gradually towards the max and min levels. Since the quantization errors is directly proportional to the step size, Δ, reducing Δ near zero would reduce errors at this region. Figure 8 shows a non uniform quantizer.
3 Binary encoding

The last step is to assign each quantization level a unique binary code to represent that level. The number of bits required to represent quantization levels can be calculated using,

$$n_b = \log_2 L$$

(8)

where $L$ is the number of quantization levels.

For example, if we have 8 levels ($L=8$), then we would need three bits to represent each level uniquely as it is shown here:

$$n_b = \log_2 8 = 3 \text{ bits}$$

and these codes are:

000, 001, 010, 011, 100, 101, 110, and 111