<u>Force</u>

Force: is something which acts upon a body which is either a push or a pull.

Force is completely characterized by its **magnitude**, **direction**, and **point of application**, and therefore its vector.

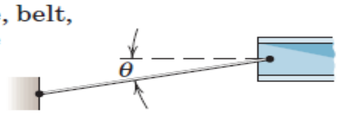
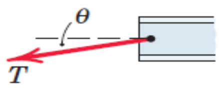
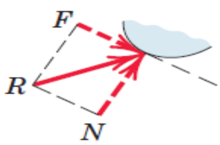
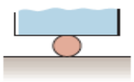
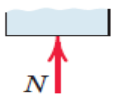
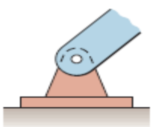
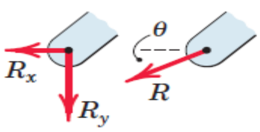
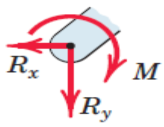
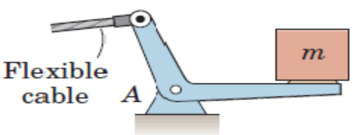
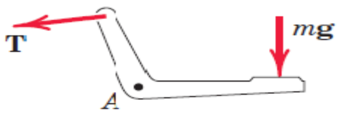
<u>Free-body diagrams</u>

A free body diagram is a sketch of the body and all the forces acting on it. They are termed fee-body diagrams because each diagram considers only the forces acting on the particular object considered.

Approach:

- Resolve force vectors in to appropriate components
- Isolate the body, remove all supports and connectors.
- Identify all EXTERNAL forces acting on the body.
- Make a sketch of the body, showing all forces acting on it.

<u>Samples of Free Body Diagrams:</u>

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope Weight of cable negligible 	 Force exerted by a flexible cable is
2. Rough surfaces 	 Rough surfaces are capable of supporting a tangential compo-nent F (frictional force) as well as a normal component N of the resultant
3. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
4. Pin connection 	Pin free to turn  Pin not free to turn  A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
5. Bell crank supporting mass m with pin support at A. 	

Force analysis:

Resolution of forces into components

It is often to decompose a single force acting at some angle from the coordinate axes into perpendicular forces called *components*. The component of a force parallel to the x-axis is called the x-component, parallel to y-axis the y-component, and so on.

These forces, when acting together, have the same external effect on a body as the original force. They are known as **components**. Finding the components of a force can be viewed as the converse of finding a resultant.

Components of a Force in XY Plane:

$$F_x = F\cos\theta_x = F\sin\theta_y$$

$$F_y = F\sin\theta_x = F\cos\theta_y$$

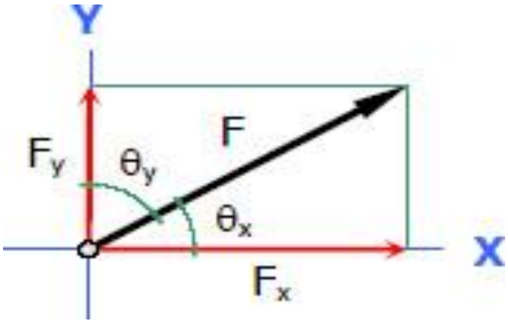
$$F = \sqrt{{F_x}^2 + {F_y}^2}$$

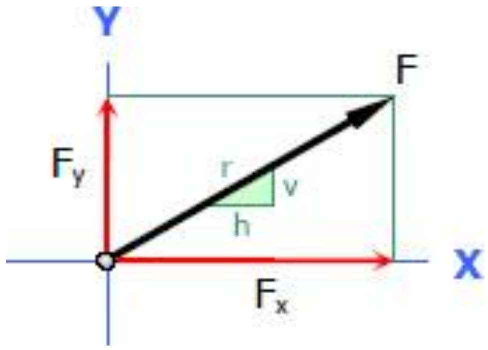
$$\tan\theta_x = \frac{F_y}{F_x}$$

$$r = \sqrt{h^2 + v^2}$$

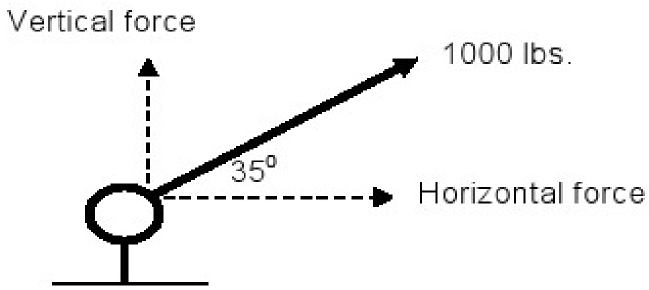
$$F_x = F(h/r)$$

$$F_y = F(v/r)$$





Most forces on inclined surfaces, or inclined forces are resolved by solving triangles. Another example of a force acting on an anchor is as follows:

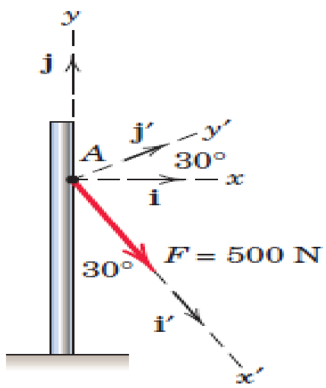


Vertical force = 1000 lbs(sin(35^0))
= **574 lbs**

Horizontal force = 1000 lbs(cos(35^0))
= **819 lbs**

Example:

The 500-N force **F** is applied to the vertical pole as shown. (1) Write **F** in terms of the unit vectors **i** and **j** and identify both its vector and scalar components. (2) Determine the scalar components of the force vector **F** along the x'- and y'-axes. (3) Determine the scalar components of **F** along the x- and y'-axes.



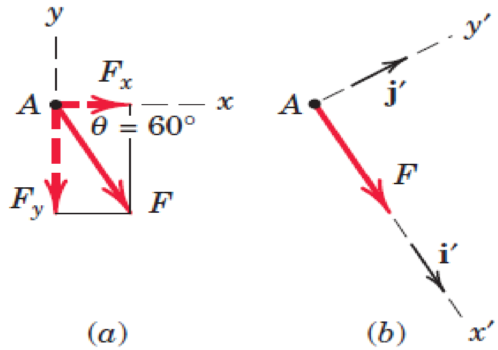
Solution:

Part (1). From Fig. a we may write **F** as

$$\bar{\mathbf{F}} = (F\cos\theta)\mathbf{i} - (F\sin\theta)\mathbf{j}$$

$$= (500\cos 60°)\mathbf{i} - (500\sin 60°)\mathbf{j}$$

$$= (250\mathbf{i} - 433\mathbf{j})\text{ N} \qquad \textit{Ans.}$$



(a) (b)

The scalar components are $F_x = 250$ N and $F_y = -433$ N. The vector components are $\mathbf{F}_x = 250\mathbf{i}$ N and $\mathbf{F}_y = -433\mathbf{j}$ N.

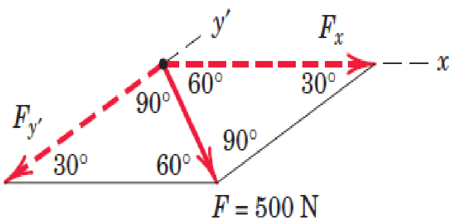
Part (2). From Fig. b we may write $\bar{\mathbf{F}}$ as $\bar{\mathbf{F}} = 500\mathbf{i}'$ N, so that the required scalar components are

$$F_{x'} = 500\text{ N} \qquad F_{y'} = 0 \qquad \textit{Ans.}$$

Part (3). The components of $\bar{\mathbf{F}}$ in the x- and y'-directions are nonrectangular and are obtained by completing the parallelogram as shown in Fig. c. The magnitudes of the components may be calculated by the law of sines. Thus,

$$\frac{|F_x|}{\sin 90°} = \frac{500}{\sin 30°} \qquad |F_x| = 1000\text{ N}$$

$$\frac{|F_{y'}|}{\sin 60°} = \frac{500}{\sin 30°} \qquad |F_{y'}| = 866\text{ N}$$



(c)

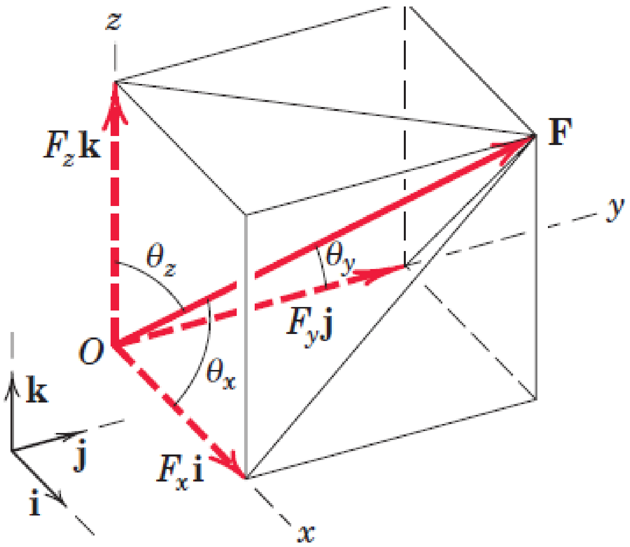
The required scalar components are then

$$F_x = 1000\text{ N} \qquad F_{y'} = -866\text{ N} \qquad \textit{Ans.}$$

Components Force in 3D Space:

Many problems in mechanics require analysis in three dimensions, and for such problems it is often necessary to resolve a force into its three mutually perpendicular components. The force **F** acting at point O in Fig. 2/16 has the *rectangular components* F_x, F_y, F_z, where

$$F_x = F\cos\theta_x \qquad F = \sqrt{F_x^{\,2} + F_y^{\,2} + F_z^{\,2}}$$
$$F_y = F\cos\theta_y$$
$$F_z = F\cos\theta_z \qquad \mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$



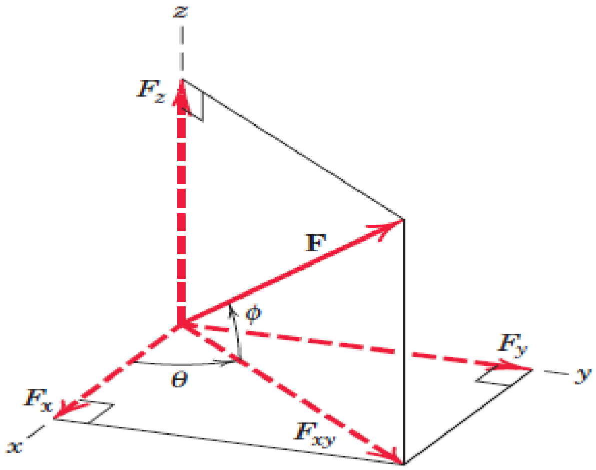
Specification by two angles which orient the line of action of the force.

Consider the geometry of the fig. We assume that the angles θ and ϕ are known. First resolve **F** into horizontal and vertical components.

$$F_{xy} = F\cos\phi$$
$$F_z = F\sin\phi$$

Then resolve the horizontal component F_{xy} into x- and y-components.

$$F_x = F_{xy}\cos\theta = F\cos\phi\cos\theta$$
$$F_y = F_{xy}\sin\theta = F\cos\phi\sin\theta$$



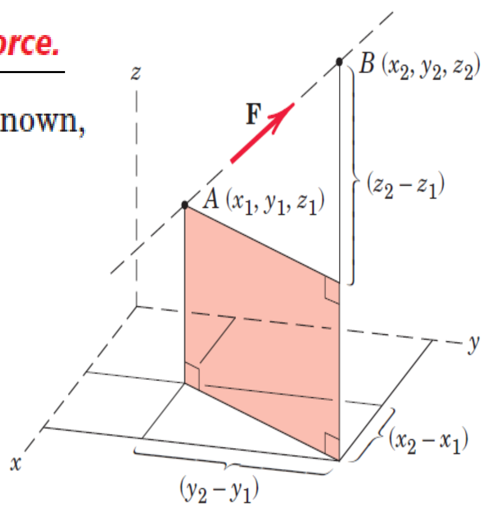
Specification by two points on the line of action of the force.

If the coordinates of points A and B of Fig the figure are known, the force **F** may be written as

$$\mathbf{F} = F\mathbf{n}_F = F\frac{\overrightarrow{AB}}{AB} = F\frac{(x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

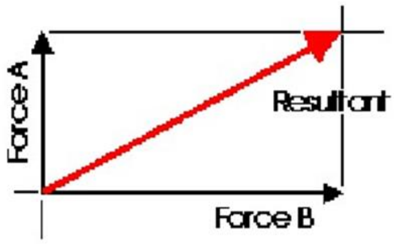
Where: $\bar{F}$: Vector

F: The magnitude of the vector



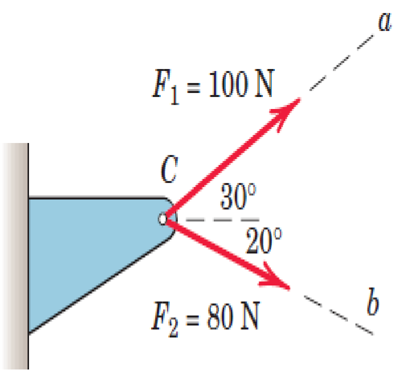
Resultant of force system:

The resultant is a representative force which has the same effect on the body as the group of forces it replaces. One can progressively resolve pairs or small groups of forces into resultants. Then another resultant of the resultants can be found and so on until all of the forces have been combined into one force. Resultants can be determined both graphically and algebraically.

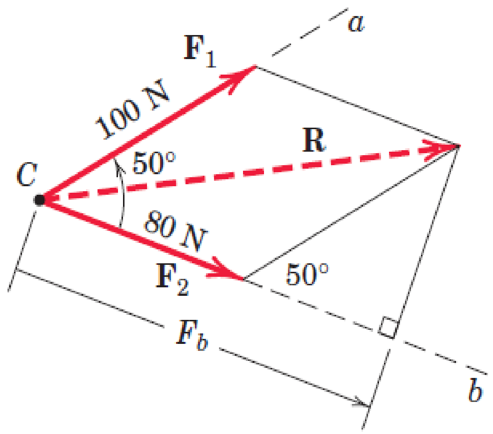


Example:

Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the bracket as shown. Determine the projection F_b of their resultant $\mathbf{R}$ onto the b-axis.



Solution:



The parallelogram addition of $\mathbf{F}_1$ and $\mathbf{F}_2$ is shown in the figure. Using the law of cosines gives us

$$R^2 = (80)^2 + (100)^2 - 2(80)(100)\cos 130^\circ \qquad R = 163.4\ \text{N}$$

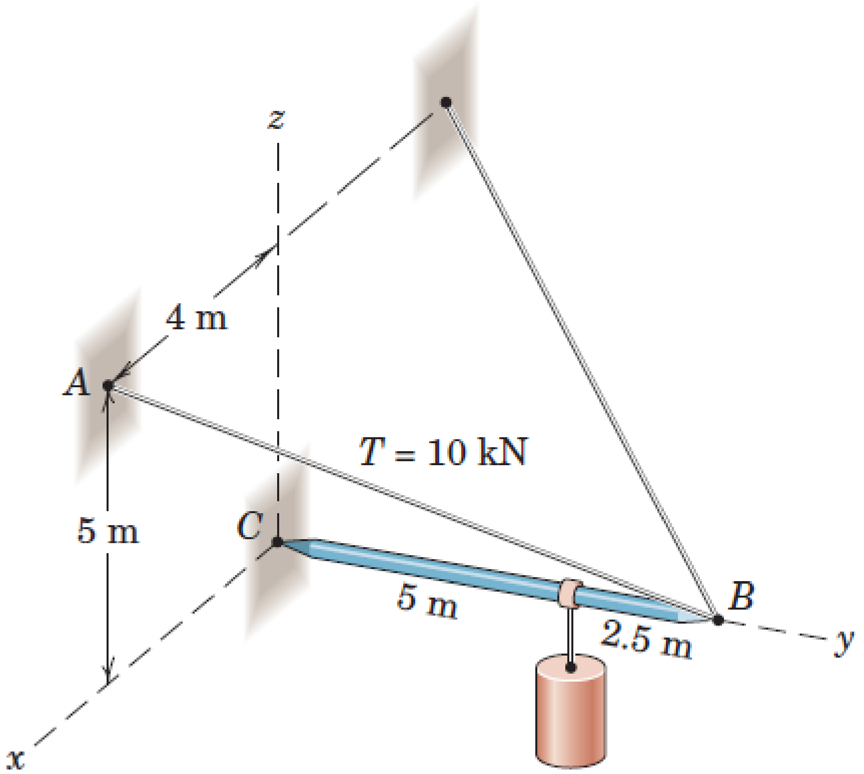
The figure also shows the orthogonal projection $\mathbf{F}_b$ of $\mathbf{R}$ onto the b-axis. Its length is

$$F_b = 80 + 100\cos 50^\circ = 144.3\ \text{N} \qquad \textit{Ans.}$$

Note that the components of a vector are in general not equal to the projections of the vector onto the same axes. If the a-axis had been perpendicular to the b-axis, then the projections and components of $\mathbf{R}$ would have been equal.

<u>Example:</u>

The tension in the supporting cable AB is 10 kN. Write the force which the cable exerts on the boom BC as a vector $\mathbf{T}$. Determine the angles θ_x, θ_y, and θ_z which the line of action of $\mathbf{T}$ forms with the positive x-, y-, and z-axes.



Solution:

$$\underline{T} = T\,\underline{n}_{AB} = 10\left[\frac{4\underline{i} - 7.5\underline{j} + 5\underline{k}}{(4^2 + (-7.5)^2 + 5^2)^{1/2}}\right]$$

$$= 10\,(0.406\,\underline{i} - 0.761\,\underline{j} + 0.507\,\underline{k})\ \text{kN} \qquad \text{Ans.}$$

$$\cos\theta_x = 0.406, \quad \theta_x = 66.1^\circ \qquad \text{Ans.}$$

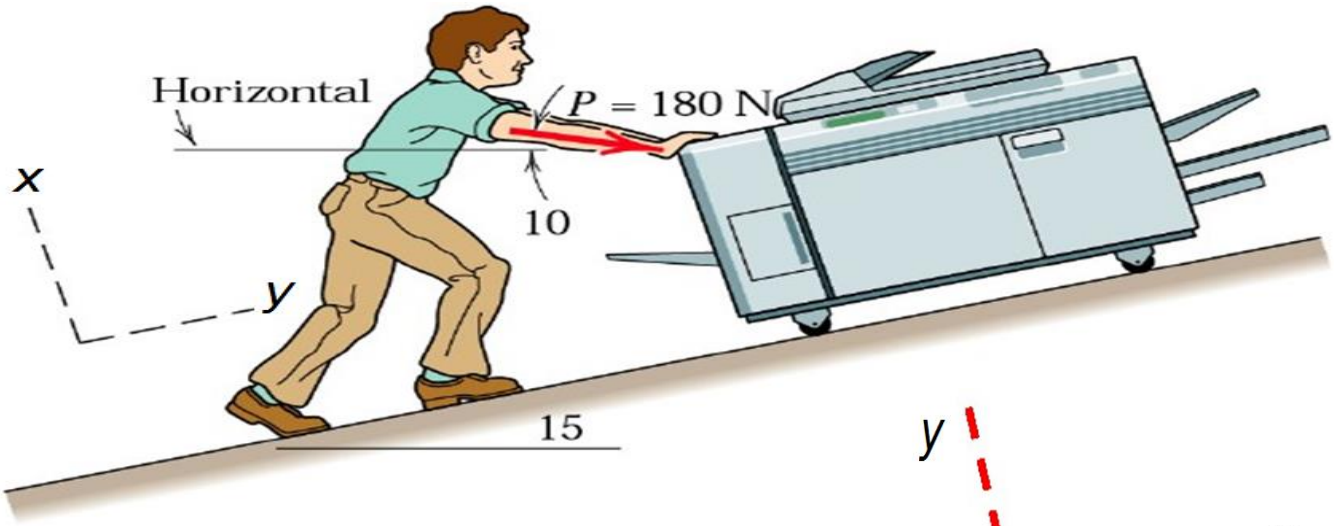
$$\cos\theta_y = -0.761, \quad \theta_y = 139.5^\circ \qquad \text{Ans.}$$

$$\cos\theta_z = 0.507, \quad \theta_z = 59.5^\circ \qquad \text{Ans.}$$

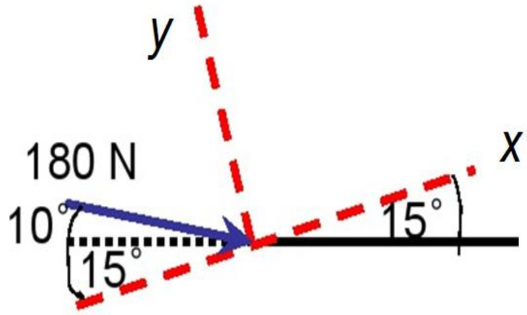
Examples with solutions

Example:

While steadily pushing the machine up an incline, a person exerts a 180 N force P as shown. Determine the components of P which are parallel and perpendicular to the incline.



Solution:

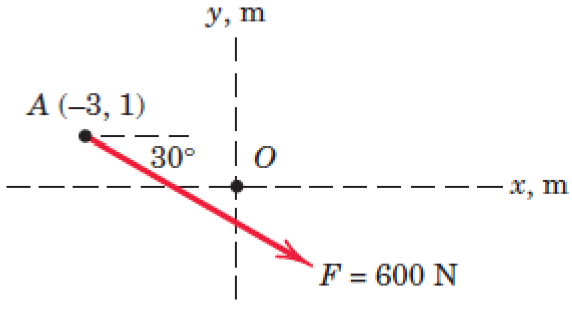


$$P_x = 180\cos(10+15) = 163.1 \text{ N}$$

$$P_y = -180\sin(10+15) = -76.1 \text{ N}$$

Example:

The magnitude of the force **F** is 600 N. Express **F** as a vector in terms of the unit vectors **i** and **j**. Identify both the scalar and vector components of **F**.



Solution:

$$\bar{F} = 600(\cos 30° i - \sin 30° j)$$

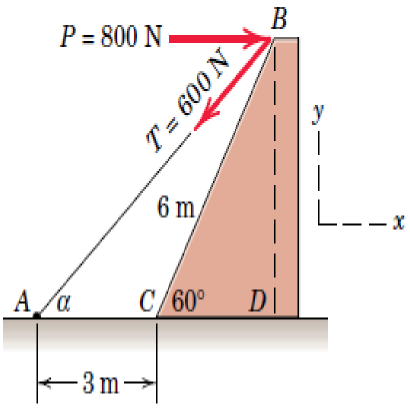
$$= 520i - 300j \text{ N} \qquad \text{Ans.}$$

Scalar components: $\begin{cases} F_x = 520 \text{ N} \\ F_y = -300 \text{ N} \end{cases}$ Ans.

Vector components: $\begin{cases} \bar{F}_x = 520i \text{ N} \\ \bar{F}_y = -300j \text{ N} \end{cases}$ Ans.

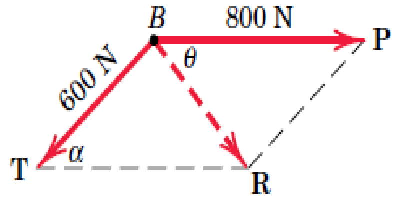
Example:

Combine the two forces **P** and **T**, which act on the fixed structure at B, into a single equivalent force **R**.



Graphical solution. The parallelogram for the vector addition of forces **T** and **P** is constructed as shown in Fig. *a*. The scale used here is 1 cm = 400 N; a scale of 1 cm = 100 N would be more suitable for regular-size paper and would give greater accuracy. Note that the angle α must be determined prior to construction of the parallelogram. From the given figure

$$\tan\alpha = \frac{\overline{BD}}{\overline{AD}} = \frac{6\sin 60^\circ}{3+6\cos 60^\circ} = 0.866 \qquad \alpha = 40.9^\circ$$

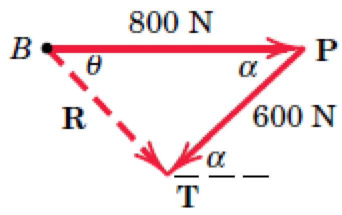


(*a*)

Measurement of the length R and direction θ of the resultant force **R** yields the approximate results

$$R = 525 \text{ N} \qquad \theta = 49^\circ \qquad \textit{Ans.}$$

Geometric solution. The triangle for the vector addition of **T** and **P** is shown in Fig. *b*. The angle α is calculated as above. The law of cosines gives



(*b*)

$$R^2 = (600)^2 + (800)^2 - 2(600)(800)\cos 40.9^\circ = 274{,}300$$

$$R = 524 \text{ N} \qquad \textit{Ans.}$$

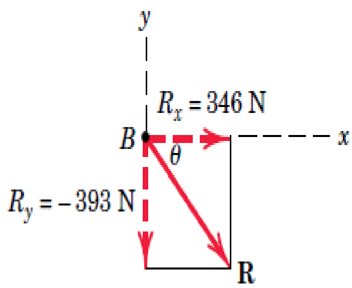
From the law of sines, we may determine the angle θ which orients **R**. Thus,

$$\frac{600}{\sin\theta} = \frac{524}{\sin 40.9^\circ} \qquad \sin\theta = 0.750 \qquad \theta = 48.6^\circ \qquad \textit{Ans.}$$

Algebraic solution. By using the x-y coordinate system on the given figure, we may write

$$R_x = \Sigma F_x = 800 - 600\cos 40.9^\circ = 346 \text{ N}$$

$$R_y = \Sigma F_y = -600\sin 40.9^\circ = -393 \text{ N}$$



(*c*)

The magnitude and dipection of the resultant force **R** as shown in Fig. *c* are then

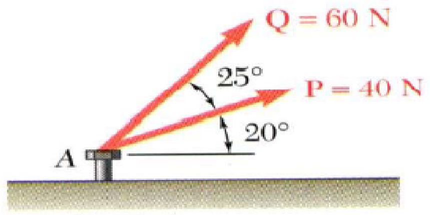
$$R = \sqrt{R_x{}^2 + R_y{}^2} = \sqrt{(346)^2 + (-393)^2} = 524 \text{ N} \qquad \textit{Ans.}$$

$$\theta = \tan^{-1}\frac{|R_y|}{|R_x|} = \tan^{-1}\frac{393}{346} = 48.6^\circ \qquad \textit{Ans.}$$

The resultant **R** may also be written in vector notation as

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} = 346\mathbf{i} - 393\mathbf{j} \text{ N} \qquad \textit{Ans.}$$

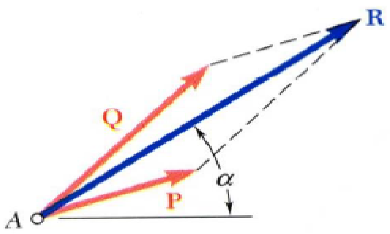
Example:



The two forces act on a bolt at *A*. Determine their resultant.

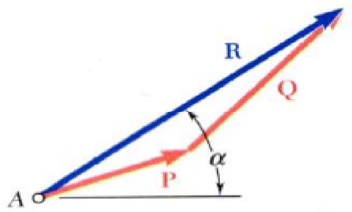
SOLUTION:

- Graphical solution - construct a parallelogram with sides in the same direction as **P** and **Q** and lengths in proportion. Graphically evaluate the resultant which is equivalent in direction and proportional in magnitude to the the diagonal.
- Trigonometric solution - use the triangle rule for vector addition in conjunction with the law of cosines and law of sines to find the resultant.



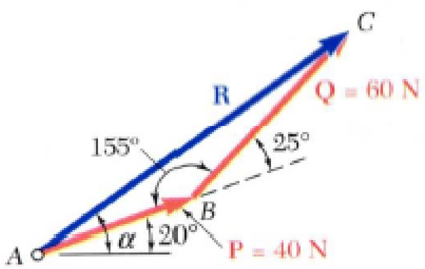
- Graphical solution - A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured,

$$\boxed{\mathbf{R} = 98\ \mathrm{N} \quad \alpha = 35°}$$



- Graphical solution - A triangle is drawn with **P** and **Q** head-to-tail and to scale. The magnitude and direction of the resultant or of the third side of the triangle are measured,

$$\boxed{\mathbf{R} = 98\ \mathrm{N} \quad \alpha = 35°}$$



- Trigonometric solution - Apply the triangle rule.

  From the Law of Cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$= (40\mathrm{N})^2 + (60\mathrm{N})^2 - 2(40\mathrm{N})(60\mathrm{N})\cos 155°$$

$$\boxed{R = 97.73\mathrm{N}}$$

From the Law of Sines,

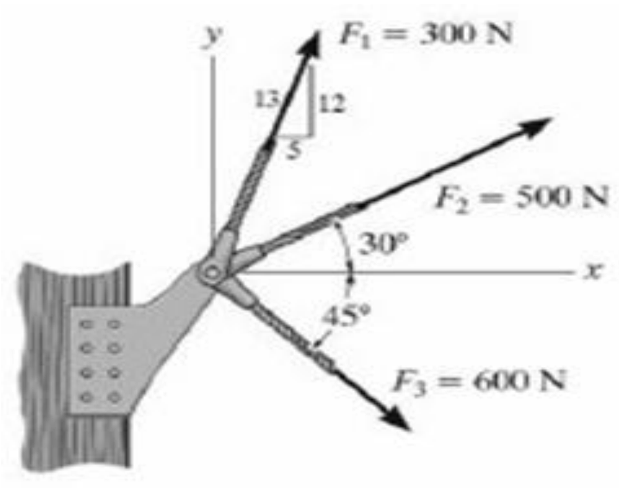
$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$

$$\sin A = \sin B \frac{Q}{R}$$
$$= \sin 155° \frac{60\mathrm{N}}{97.73\mathrm{N}}$$
$$A = 15.04°$$
$$\alpha = 20° + A$$
$$\boxed{\alpha = 35.04°}$$

Example:

Given: Three concurrent forces acting on a bracket

Find: The magnitude and angle of the resultant force.



$$\boldsymbol{F_1} = \{ (5/13)\,300\,\boldsymbol{i} + (12/13)\,300\,\boldsymbol{j} \}\ \text{N}$$

$$= \{ 115.4\ \boldsymbol{i} + 276.9\ \boldsymbol{j} \}\ \text{N}$$

$$\boldsymbol{F_2} = \{500 \cos(30°)\,\boldsymbol{i} + 500 \sin(30°)\,\boldsymbol{j} \}\ \text{N}$$

$$= \{ 433.0\ \boldsymbol{i} + 250\ \boldsymbol{j} \}\ \text{N}$$

$$\boldsymbol{F_3} = \{ 600 \cos(45°)\ \boldsymbol{i} - 600 \sin(45°)\ \boldsymbol{j} \}\ \text{N}$$

$$\{ 424.3\ \boldsymbol{i} - 424.3\ \boldsymbol{j} \}\ \text{N}$$

Summing up all the $\boldsymbol{i}$ and $\boldsymbol{j}$ components respectively, we get,

$$\boldsymbol{F_R} = \{ (115.4 + 433.0 + 424.3)\,\boldsymbol{i} + (276.9 + 250 - 424.3)\,\boldsymbol{j} \}\text{N}$$

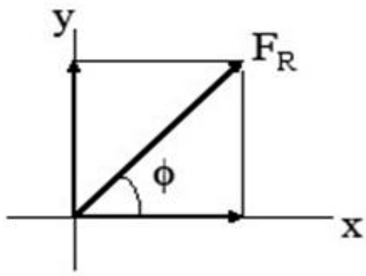
$$= \{ 972.7\,\boldsymbol{i} + 102.7\,\boldsymbol{j} \}\ \text{N}$$

Now find the magnitude and angle,

$$F_R = ((972.7)^2 + (102.7)^2)^{½} = 978.1\ \text{N}$$

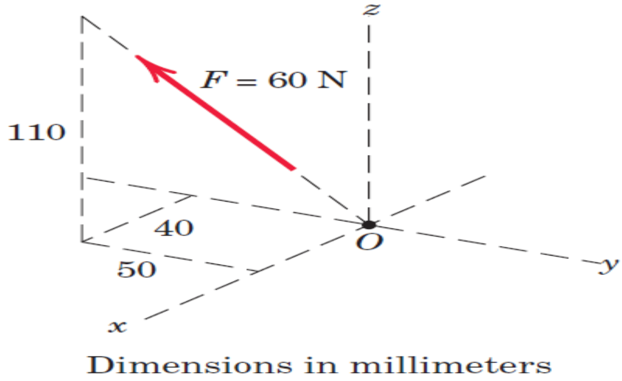
$$\phi = \tan^{-1}(102.7 / 972.7) = 6.03°$$

From Positive x axis, $\phi = 6.03°$



Example:

Express **F** as a vector in terms of the unit vectors **i**, **j**, and **k**. Determine the angle between **F** and the *y*-axis.



Solution:

$$\bar{F} = F\bar{n}$$

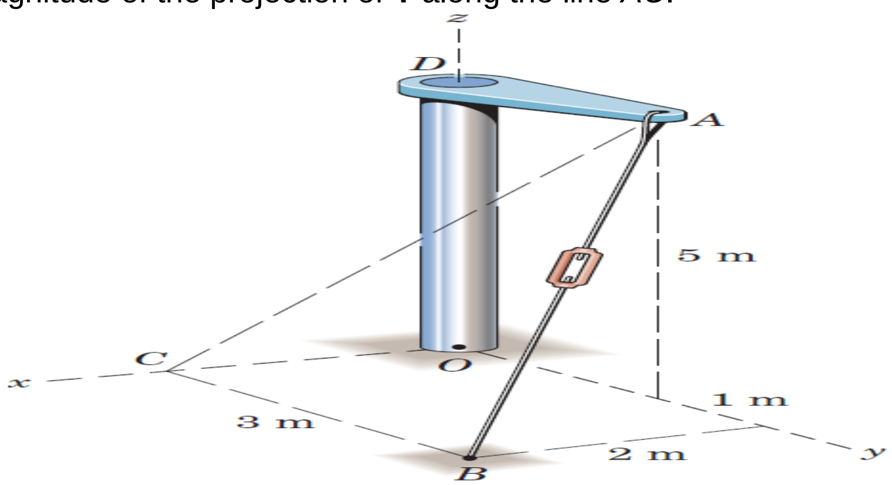
$$= 60\left[\frac{40i - 50j + 110k}{\sqrt{40^2 + 50^2 + 110^2}}\right]$$

$$= 18.86i - 23.6j + 51.9k \quad N$$ **Ans.**

$$\cos\theta_y = \frac{F_y}{F} = \frac{-23.6}{60}, \quad \theta_y = 113.1^\circ$$ **Ans.**

Example:

The turnbuckle is tightened until the tension in the cable *AB* equals 2.4 kN. Determine the vector expression for the tension **T** as a force acting on member *AD*. Also find the magnitude of the projection of **T** along the line *AC*.



Solution:

$$\bar{T} = T\bar{n}_{AB} = 2.4\left(\frac{2i + j - 5k}{\sqrt{2^2 + 1^2 + 5^2}}\right)$$

$$= 0.876i + 0.438j - 2.19k \quad kN$$ **Ans.**

$$\text{Projection} \quad T_{AC} = T \cdot \bar{n}_{AC}$$

$$= (0.876i + 0.438j - 2.19k) \cdot \left(\frac{2i - 2j - 5k}{\sqrt{2^2 + 2^2 + 5^2}}\right)$$

$$= 2.06 \quad kN$$ **Ans.**