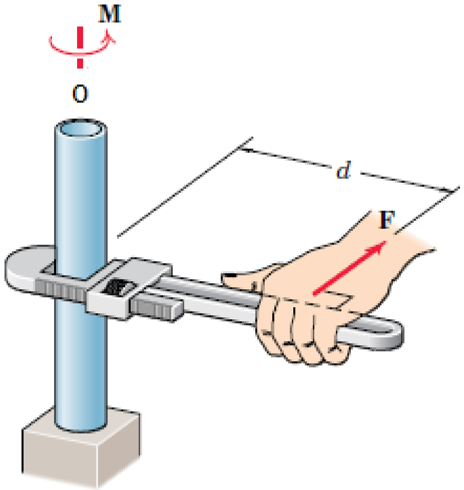
Moment, Coples and Equlibrium

Moment of a force:

The Moment of Force (F) about an axis through Point (o) or for short is the product of the magnitude of the force and the perpendicular distance between point (o) and the line of action of force (F)



$$M = F\,d$$

The units of a Moment are: *N·m* in the SI system

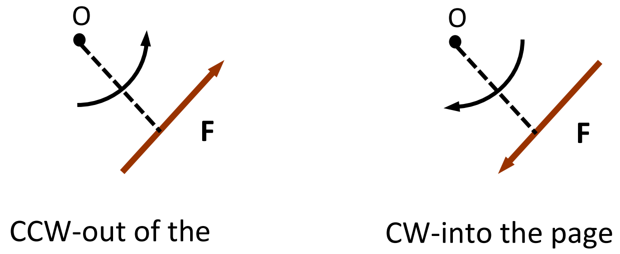
ft·lbs or *in·lbs* in the US Custor

M: Magnitude of the moment of **F** around point O

d: Perpendicular distance from O to the line of action of **F**

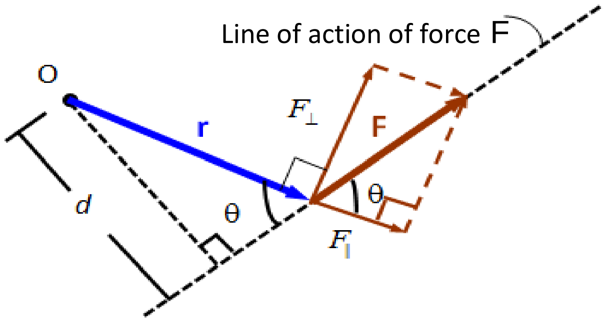
Direction of the moment in 2-D:

The direction of the moment is given by the right hand rule: Counter Clockwise (CCW) is out of the page, Clockwise (CW) is into the page.



Moving a force along its line of action:

Moving a force along its line of action, results in a new force system which is equivalent to the original force system.

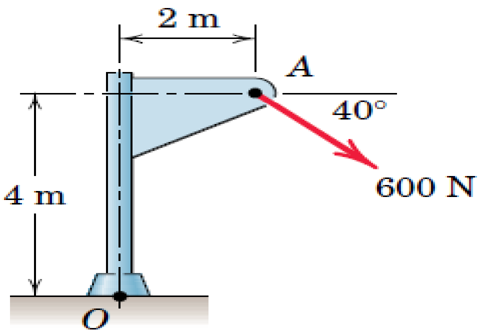


Note: moving a force along its line of action does not change its moment

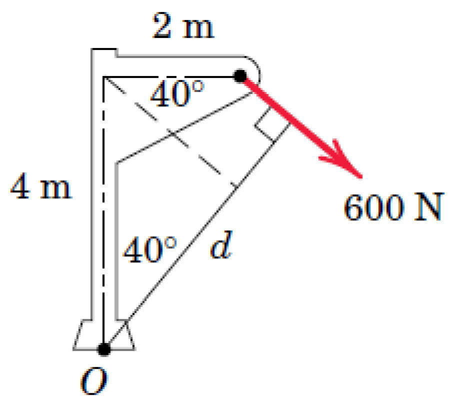
$$M = Fd = Fr\,\text{Sin}\,(\theta)$$

Example:

Calculate the magnitude of the moment about the base point O of the 600-N force in five different ways.



Solution:



(I) The moment arm to the 600-N force is

$$d = 4\cos 40° + 2\sin 40° = 4.35 \text{ m}$$

By $M = Fd$ the moment is clockwise and has the magnitude

$$M_O = 600(4.35) = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

(II) Replace the force by its rectangular components at A

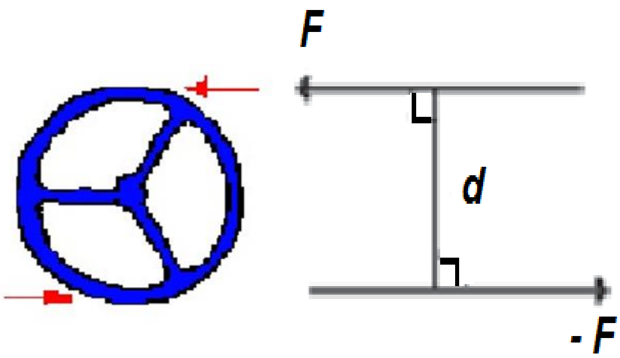
$$F_1 = 600\cos 40° = 460 \text{ N}, \qquad F_2 = 600\sin 40° = 386 \text{ N}$$

the moment becomes

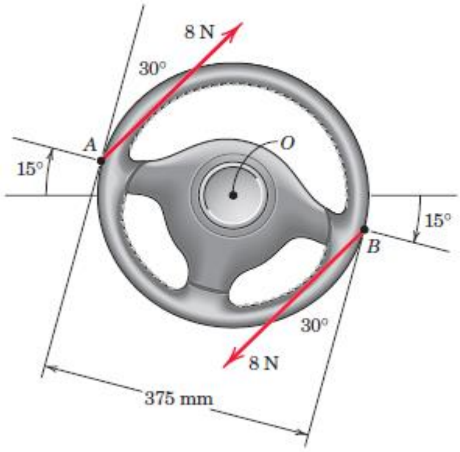
$$M_O = 460(4) + 386(2) = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

Moment of a Couple:

A couple can be defined as two parallel forces, having the same magnitude, opposite directions and separated by a perpendicular distance *d*



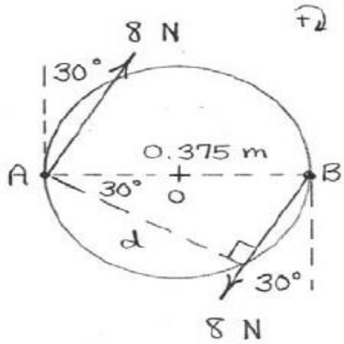
The moment of a couple is the product of the magnitude of one of the forces and the perpendicular distance between their lines of action. **M = F x d.** It has the units of kip-feet, pound-inches, KN-meter, etc.



Example:

During a steady right turn, a person exerts the forces shown on the steering wheel. Note that each force consists of a tangential component and a radially inward component. Determine the moment exerted about the steering column at *O*.

Solution:

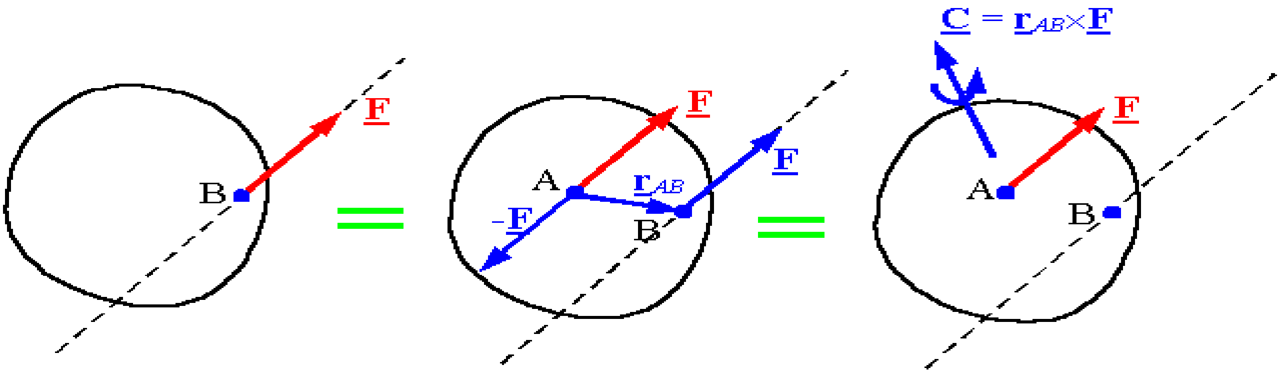


$$\curvearrowright M = Fd = 8\,(0.375\cos 30^\circ)$$

$$= \underline{2.60\ \text{N}\cdot\text{m}\ \ \text{CW}}$$

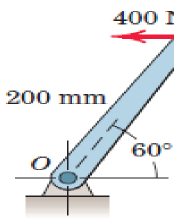
Moving a force off its line of action:

If a force is moved off its line of action, a couple must be added to the force system so that the new system generates the same moment as the old system.

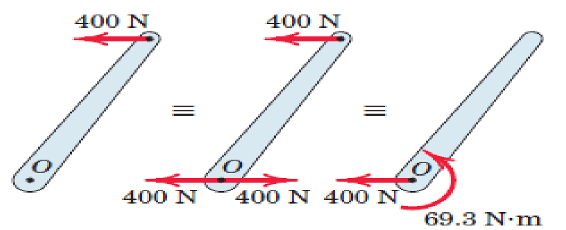


Example:

Replace the horizontal 400-N force acting on the lever by an equivalent system consisting of a force at *O* and a couple.



Solution:



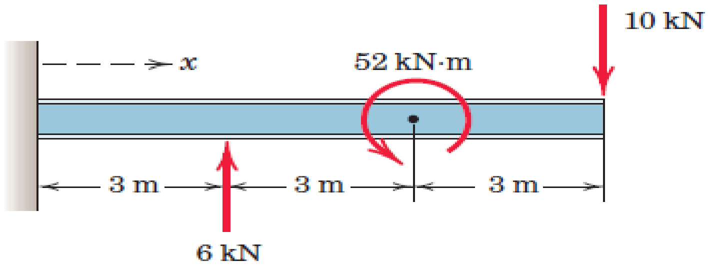
We apply two equal and opposite 400-N forces at *O* and identify the counterclockwise couple

$$[M = Fd] \qquad M = 400(0.200\sin 60^\circ) = 69.3\ \text{N}\cdot\text{m} \qquad \textit{Ans.}$$

Thus, the original force is equivalent to the 400-N force at *O* and the 69.3-N·m couple as shown in the third of the three equivalent figures.

Example:

Determine and locate the resultant **R** of the two forces and one couple acting on the I-beam.

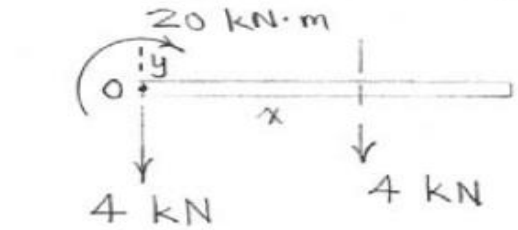


Solution:

Force-Couple system at point O:

$$\underline{R} = \Sigma \underline{F} = (6-10)\,\underline{j} = -4\,\underline{j} \ \text{kN}$$

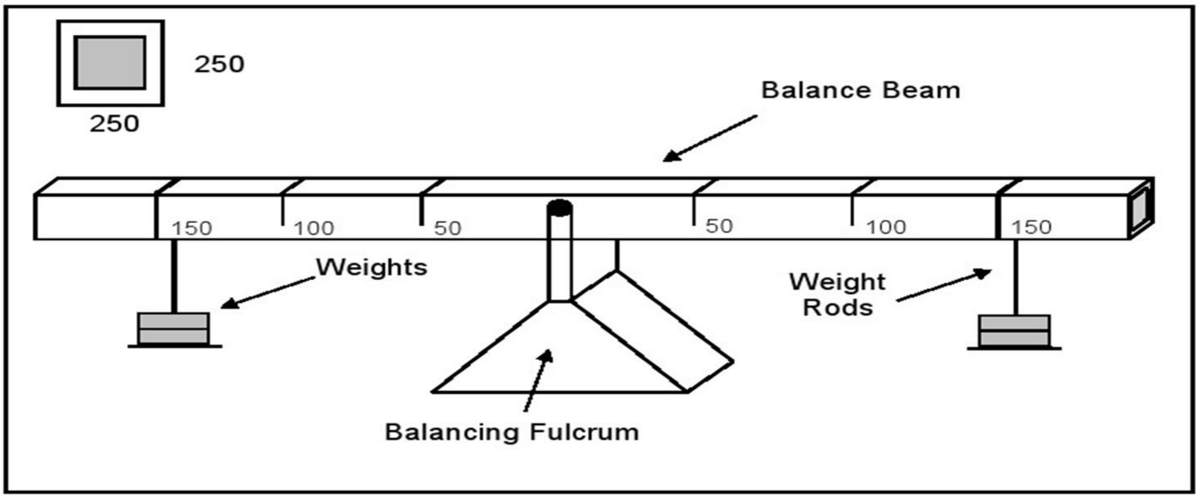
$$\circlearrowleft + \; M_O = 6(3) - 10(9) + 52 = -20 \ \text{kN·m}$$



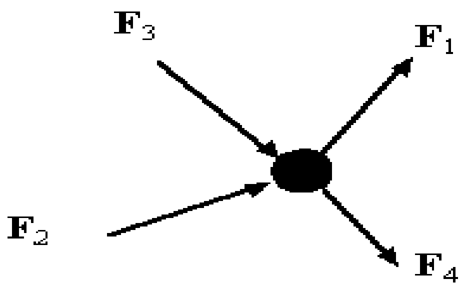
$$x = \frac{M_O}{R} = \frac{20}{4} = \underline{5 \ \text{m}} \quad (\text{on beam!})$$

Equilibrium:

A body is in equilibrium if the resultant of all the external forces and moments acting on the body is zero**.**



$$\sum \mathbf{F} = \mathbf{0}$$
$$\sum \mathbf{M} = \mathbf{0}$$



Equilibrium equations in component form:

In a rectangular coordinate system the equilibrium equations can be represented by three scalar equations:

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum F_z = 0$$

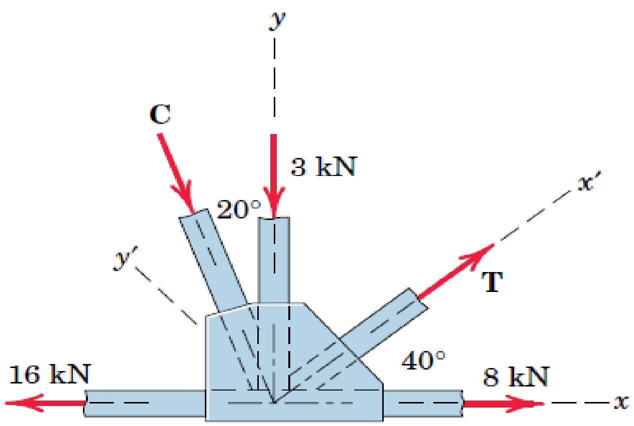
Sum of All Moments (Mz) = 0

(Sum of All Moments (My) = 0)

(Sum of All Moments (Mx) = 0)

Example:

Determine the magnitudes of the forces **C** and **T**, which, along with the other three forces shown, act on the bridge-truss joint.



The given sketch constitutes the free-body diagram of the isolated section of the joint in question and shows the five forces which are in equilibrium.

For the x-y axes as shown we have

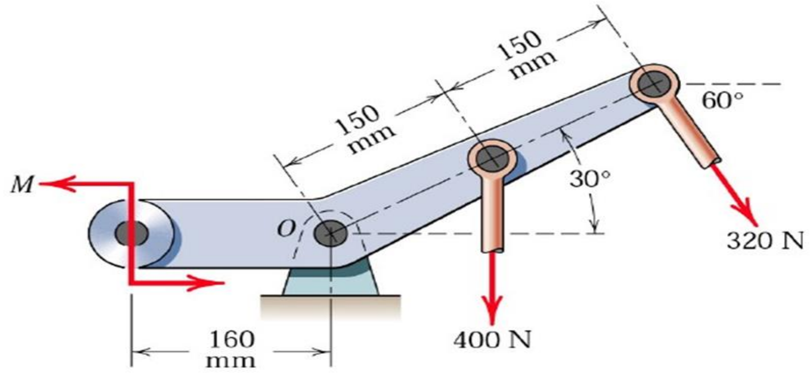
$$[\Sigma F_x = 0] \qquad 8 + T\cos 40^\circ + C\sin 20^\circ - 16 = 0$$
$$0.766T + 0.342C = 8 \qquad (a)$$
$$[\Sigma F_y = 0] \qquad T\sin 40^\circ - C\cos 20^\circ - 3 = 0$$
$$0.643T - 0.940C = 3 \qquad (b)$$

Simultaneous solution of Eqs. (a) and (b) produces

$$T = 9.09 \text{ kN} \qquad C = 3.03 \text{ kN} \qquad Ans.$$

Examples & Solutions

Example: If the resultant of the two forces and couple *M* passes through point *O*, determine *M*.



Solution:

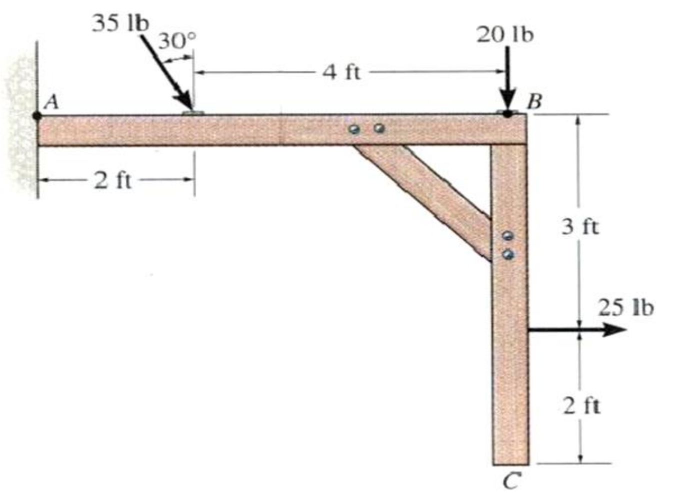
$$M_O = M - 400 \times 0.15\cos 30 - 320 \times 0.3 = 0$$

$$M = 148 \text{ Nm CCW}$$ Ans.

Example:

Given: A 2-D force and couple system as shown.

Find: The equivalent resultant force and couple moment acting at A.



Solution

1) Sum all the x and y components of the forces to find F_{RA}.

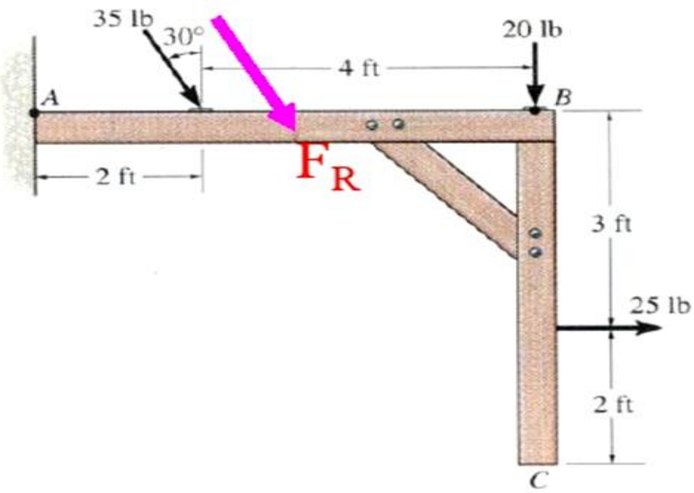
2) Find and sum all the moments resulting from moving each force to A.

$$+\rightarrow \Sigma F_{Rx} = 25 + 35 \sin 30° = 42.5 \text{ lb}$$

$$+\downarrow \Sigma F_{Ry} = 20 + 35 \cos 30° = 50.31 \text{ lb}$$

$$+\circlearrowleft M_{RA} = 35 \cos 30° (2) + 20(6) - 25(3)$$

$$= 105.6 \text{ lb·ft}$$

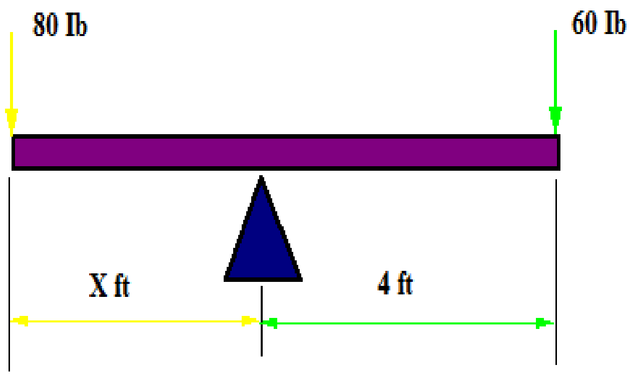


$$F_R = (42.5^2 + 50.31^2)^{1/2} = 65.9 \text{ lb}$$

$$\theta = \tan^{-1}(50.31/42.5) = 49.8°$$

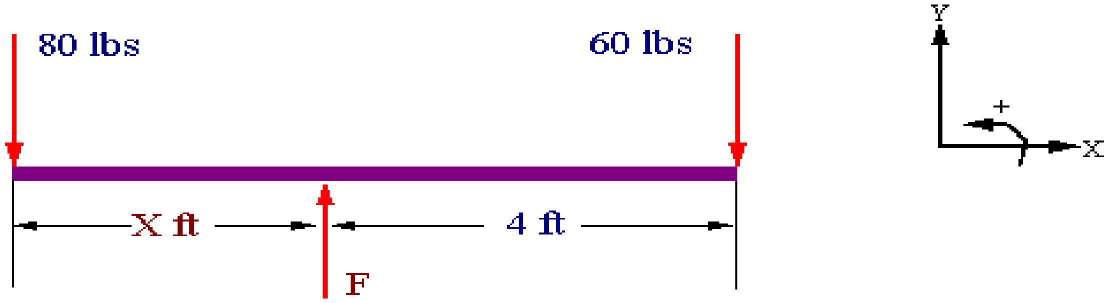
Example:

Two children balance a see-saw in horizontal equilibrium. One weighs 80 pounds, and the other weighs 60 pounds and is sitting 4 ft. from the fulcrum. Find the force the fulcrum applies to the beam and the distance to the fulcrum to the 80 lb. child. (Neglect the mass of the see-saw.)



Solution:

Free body diagram

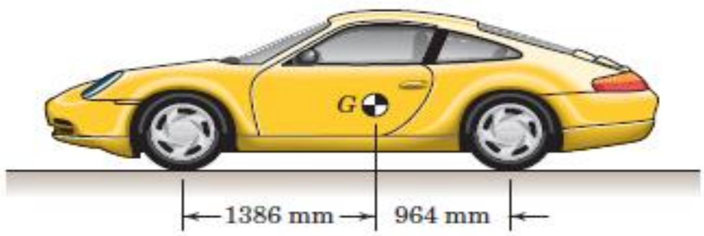


$$\sum_{\rightarrow +} F_X = 0 = \text{No X forces.} \quad \sum_{+\uparrow} F_Y = 0 = -80 + -60 + F \quad \Rightarrow \quad \underline{F = 140 \text{ lbs}}\uparrow$$

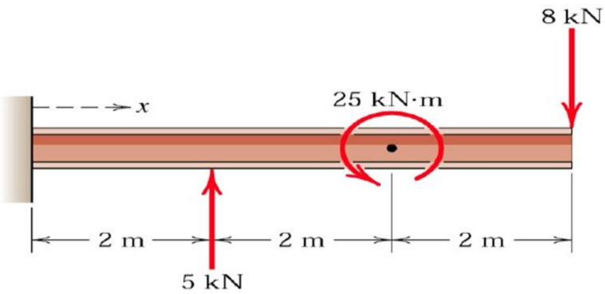
$$\sum_{+\circlearrowleft} M_{Fulcrum} = 0 = 80 * X - 60 * 4 \quad \Rightarrow \quad \underline{X = 3 \text{ ft}}$$

Question:

The mass center *G* of the 1400-kg rear-engine car is located as shown in the figure. Determine the normal force under each tire when the car is in equilibrium. State any assumptions.

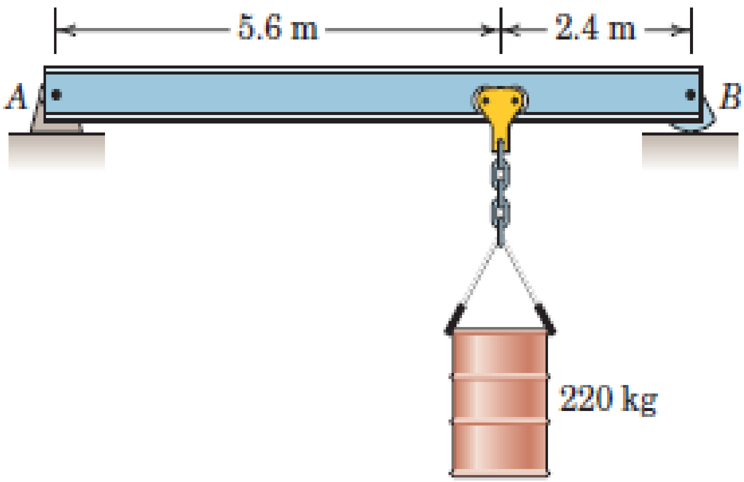


Question: Determine and locate the resultant R of two forces and one couple acting on the I beam

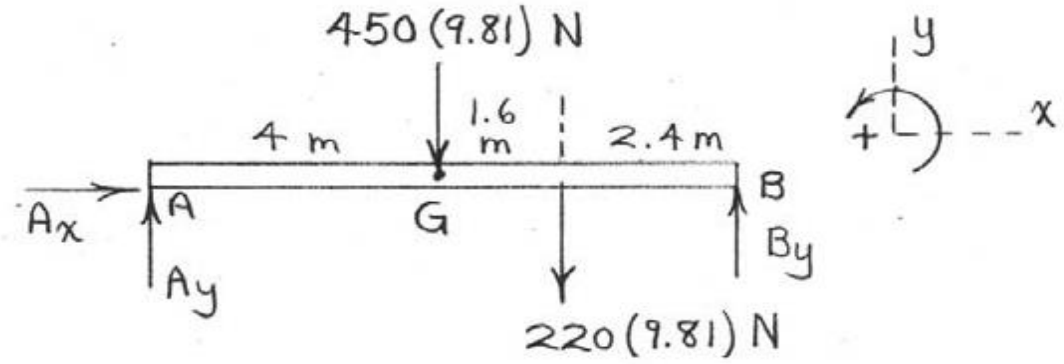


Example:

The 450-kg uniform I-beam supports the load shown. Determine the reactions at the supports.



Solution:



From $\Sigma F_x = 0, \quad A_x = 0$

$\Sigma M_A = 0: \; -450(9.81)4 - 220(9.81)(5.6)$

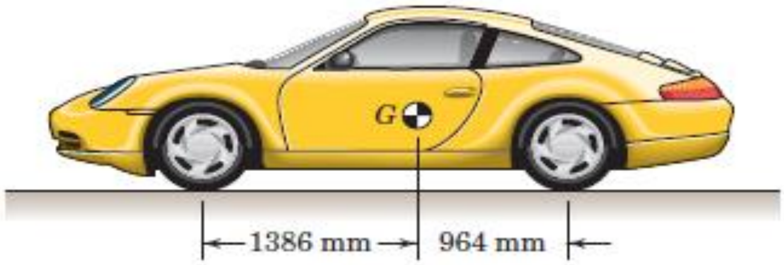
$+ B_y(8) = 0 \;, \quad B_y = 3720 \text{ N}$ Ans.

$\Sigma F_y = 0: \; A_y - 450(9.81) - 220(9.81) + 3720 = 0$

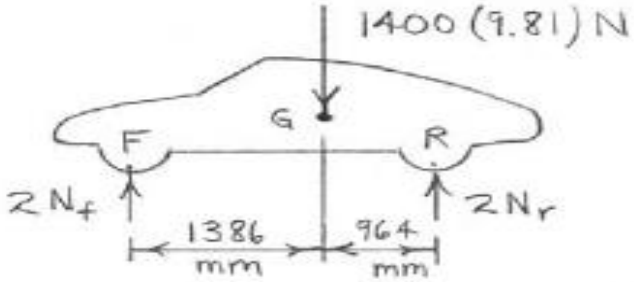
$A_y = 2850 \text{ N}$ Ans.

Question:

The mass center *G* of the 1400-kg rear-engine car is located as shown in the figure. Determine the normal force under each tire when the car is in equilibrium. State any assumptions.



Solution:



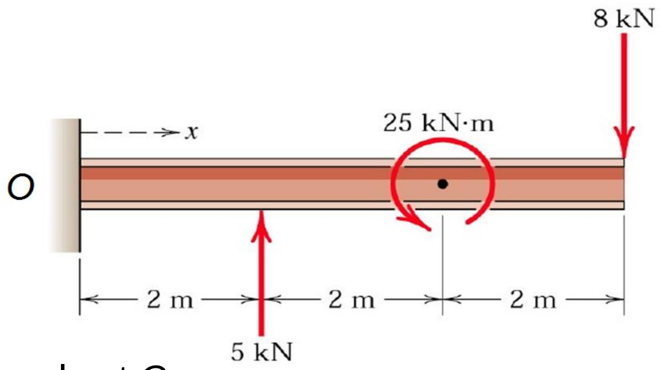
$+\uparrow \Sigma F = 0 : \quad 2N_f + 2N_r - 1400(9.81) = 0$

$\circlearrowleft+ \Sigma M_F = 0 : \quad -1400(9.81)(1386) + 2N_r(1386 + 964) = 0$

Solution : $\begin{cases} N_f = 2820 \text{ N} \\ N_r = 4050 \text{ N} \end{cases}$

Ans.

Question: Determine and locate the resultant R of two forces and one couple acting on the (**I**) beam



Solution:

First find the equivalent force-couple at *O*

R = 5 – 8 = - 3 KN downward

The moment around O is:

Mo = 5 x 2 + 25 – 8 x 6 = - 13 KN.m CW

X = *Mo* / R = 13 / 3 = 4.33 m *Ans.*