Fundamentals of Communications Engineering

Department of Communications Engineering, College of Engineering, University of Diyala, 2016-2017

Class: Second Year
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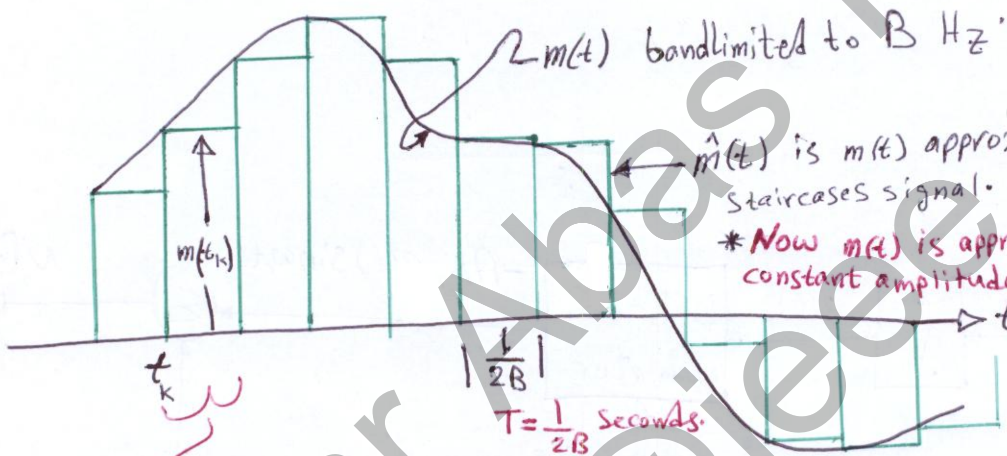
Room: Comm-02

Lecture: 18

* Wide-Band FM (WBFM) :-

* If $|k_f\, a(t)| \gg 1$: The higher order terms in Eq. (5.8) can not be ignored.

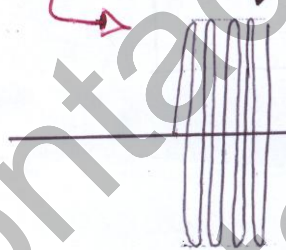
(a)



* Now m(t) is approximated by pulses of constant amplitudes.

$T = \frac{1}{2B}$ seconds.

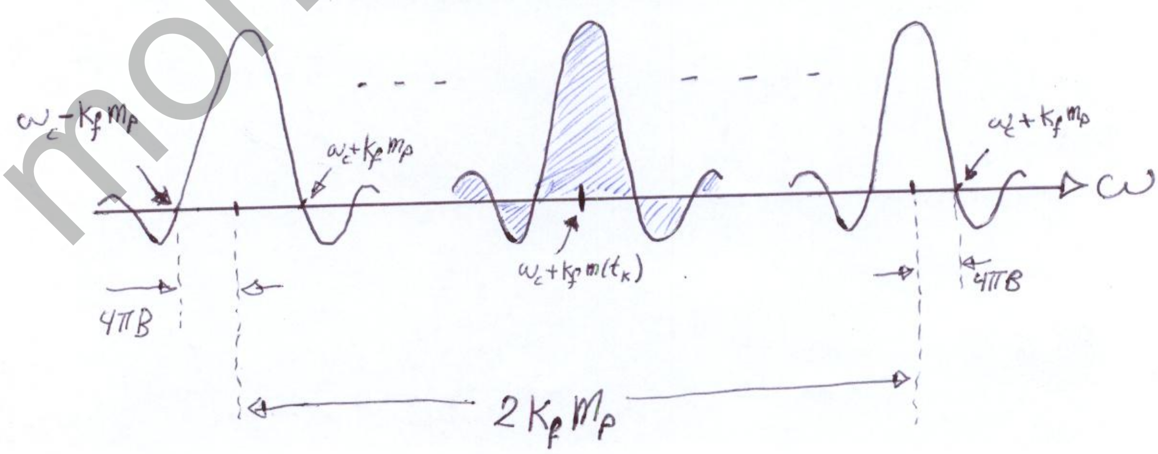
(b)



$$\omega_i = \omega_c + k_f\, m(t_k)$$

* The FM signal is a sinusoid of frequency $\omega_c + k_f m(t_k)$ and duration $T = \frac{1}{2B}$
* The FM signal for $\hat{m}(t)$ consists of a sequence of such sinusoidal pulses corresponding to various pulses of $\hat{m}(t)$.
* The FM spectrum of $\hat{m}(t)$ consists of the sum of the Fourier transforms of these sinusoidal pulses.
* The F.T.$\{\hat{m}(t)\}$ is a sinc function.

(c)



Important Notes :

* K_f is the frequency deviation constant or frequency sensitivity constant.

* K_p is the phase deviation constant or phase sensitivity constant.

* The maximum amplitude of the pulse is m_p.
* The minimum amplitude of the pulse is $-m_p$.

* The maximum frequency of the sinusoidal pulses corresponding to FM signal is $\omega_c + K_f m_p$.

* The minimum frequency of the sinusoidal pulses corresponding to FM signal is $\omega_c - K_f m_p$.

* Now, the carrier frequency deviation is :

$$\boxed{\Delta\omega = K_f m_p} \quad (5.11)$$

* Note that the maximum deviation is $2\Delta\omega$.

* Carrier frequency deviation in Hz is

$$\boxed{\Delta f = \frac{k_f m_p}{2\pi}}$$

* Carson's Rule says:

$$\boxed{B_{FM} = 2(\Delta f + B)}$$

B is the bandwidth of the message signal

OR

$$\boxed{B_{FM} = 2\left(\frac{k_f m_p}{2\pi} + B\right)} \quad (5.13)$$

* For truly wideband case: $\Delta f \gg B$ then

$$\boxed{B_{FM} \simeq 2\Delta f} \quad (5.14)$$

* Let the deviation ratio be defined as β, then

$$\boxed{\beta = \frac{\Delta f}{B}} \quad (5.15)$$

* Deviation ratio β is controles the amount of modulation

∴ Carson's Rule becomes:

$$\boxed{B_{FM} = 2B(\beta + 1)} \quad (5.16)$$

* In tone-modulated FM β is called the modulation index.

Phase Modulation :-

* All the results derived for FM can directly applied to PM.

* The instantaneous frequency is given by :-

$$\omega_i = \omega_c + k_p \dot{m}(t)$$

* Therefore, the frequency deviation $\Delta\omega$ is given by

$$\boxed{\Delta\omega = k_p m_p'} \quad (5.17a)$$

where $\boxed{m_p' = [\dot{m}(t)]_{max}}$

* Therefore,

$$\boxed{B_{PM} = 2(\Delta f + B)} \quad (5.18a)$$

OR

$$\boxed{B_{PM} = 2\left(\frac{k_p m_p'}{2\pi} + B\right)} \quad (5.18b)$$

Single-tone FM modulation :-

* consider the message signal $m(t)$ as

$$\Rightarrow \quad m(t) = \alpha \cos\omega_m t$$

Since: $a(t) = \int_{-\infty}^{t} m(\tau)\, d\tau$ —(5.6)

$$\therefore \quad a(t) = \frac{\alpha}{\omega_m} \sin\omega_m t$$

$$\hat{\varphi}_{FM}(t) = A\, e^{\left(j\omega_c t + \frac{k_f \alpha}{\omega_m} \sin\omega_m t\right)}$$

we have $\quad \Delta\omega = k_f m_p = \alpha k_f$

* Bandwidth of $m(t) = B$ Hz,
* The deviation ratio (modulation index in single-tone modulation) is

$$\boxed{\beta = \frac{\Delta f}{f_m} = \frac{\Delta\omega}{\omega_m} = \frac{\alpha k_f}{\omega_m}}$$

Hence :- $\quad \hat{\varphi}_{FM}(t) = A\, e^{j(\omega_c t + \beta \sin\omega_m t)}$

$$= A\, e^{j\omega_c t}\left[e^{j\beta \sin\omega_m t}\right] \quad \text{—(5.19)}$$

* Note that $e^{j\beta \sin \omega_m t}$ is periodic can be expanded using Fourier Series as:-

$$e^{j\beta \sin \omega_m t} = \sum_{n=-\infty}^{\infty} C_n e^{j\omega_m t}$$

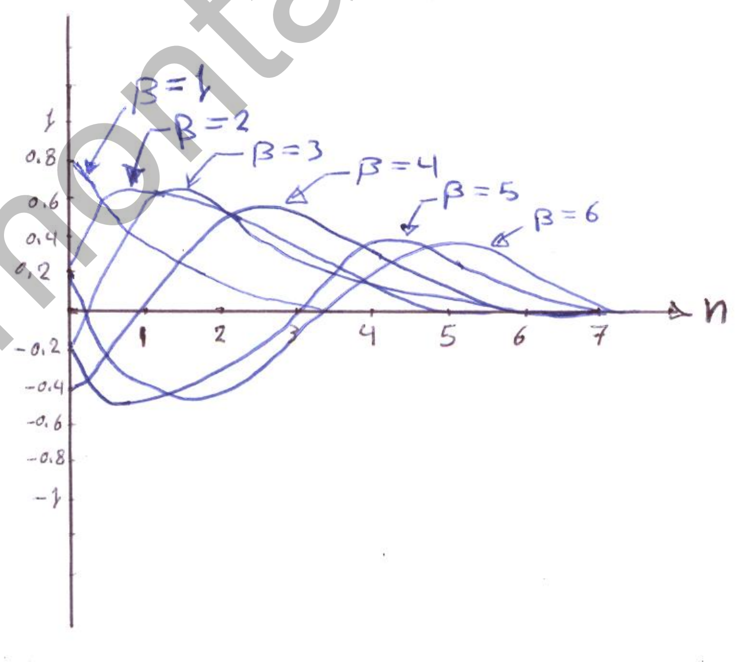
where $C_n = \frac{\omega_m}{2\pi} \int_{-\pi/\omega_m}^{\pi/\omega_m} e^{j\beta \sin \omega_m t} e^{-jn\omega_m t} dt$

Let $\omega_m t = x$, we get

$$C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)}$$

can not be evaluated in closed form. It can be Tabulated only.

* This integral is called Bessel function of the first kind and n^{th} order denoted by $J_n(\beta)$



$\therefore \quad \hat{\phi}_{FM}(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j(\omega_c t + n\omega_m t)}$

and

$$\boxed{\hat{\phi}_{FM}(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t}$$

* The modulated signal has a carrier component and infinite number of frequencies

$\omega_c \pm \omega_m$

$\omega_c \pm 2\omega_m$

$\omega_c \pm 3\omega_m$

$\omega_c \pm 4\omega_m$

$\omega_c \pm n\omega_m$

* The strength of the n^{th} sideband at $\omega_c + n\omega_m$ is $J_n(\beta)$.

* Note that $J_{-n}(\beta) = (-1)^n J_n(\beta)$

$\therefore$ the magnitude of LSB at $\omega = \omega_c - n\omega_m$ is the same as that of USB at $\omega = \omega_c + n\omega_m$

* For a given β, $J_n(\beta)$ decreases with n.
* For Large n, $J_n(\beta)$ is negligible.
* There are only a finite number of significant sidebands.
* $J_n(\beta)$ is negligible for $\boxed{n > \beta + 1}$

* since $J_n(\beta)$ is negligible when $n > \beta+1$, then,

the number of significant sidebands is $(\beta+1)$.

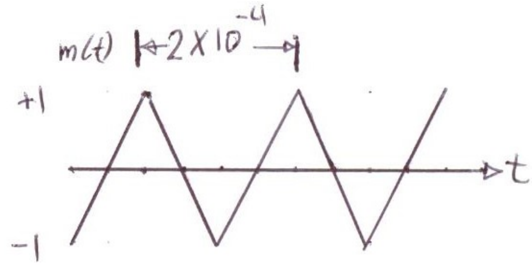
$\therefore$ Bandwidth of FM carrier is :-

$$B_{FM} = 2nf_m$$
$$= 2(\beta+1)f_m$$
$$= 2(\Delta f + B)$$

EX. 5.3 (a) Estimate B_{FM} and B_{PM} for the modulating signal $m(t)$ for the figure below when $k_f = 2\pi \times 10^5$ and $k_p = 5\pi$.

(b) Repeat the problem if the amplitude of $m(t)$ is doubled [if $m(t)$ is multiplied by 2].

Solution :-



(a) The peak amplitude of $m(t)$ is unity.

$\therefore m_p = 1$

$$\omega_0 = \frac{2\pi}{2\times10^{-4}} = 10^4\pi \text{ rad/s}$$

$$m(t) = \sum_n C_n \cos n\omega_0 t$$

$$C_n = \begin{cases} \dfrac{8}{\pi^2 n^2} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

let $n=1$: $C_1 = \frac{8}{\pi^2} = 0.81057$

$n=3$; $C_3 = \frac{8}{\pi^2} \cdot \frac{1}{9} = 0.09$

$n=5$; $C_5 = \frac{8}{\pi^2} \frac{1}{25} = 0.032$

$n=7$; $C_7 = \frac{8}{\pi^2} \frac{1}{49} = 0.0165$

$n=9$; $C_9 = \frac{8}{\pi^2} \frac{1}{81} = 0.01$

$n=11$; $C_{11} = 0.0067$

$n=13$; $C_{13} = 0.0048$

we see as long as n increases, the harmonic amplitudes decreases.

*We can evalute the bandwidth by including only three harmonics $n=3$,

$\therefore \omega_0 = 10^4\pi \text{ rad/s} = \frac{10^4\pi}{2\pi} = 5 \text{ KHz}$

$\therefore B = 3f_0 = 3 \times 5 \text{ KHz} = 15 \text{ KHz}$

Hence; For FM :-

$\Delta f = \frac{1}{2\pi} k_f m_p = \frac{1}{2\pi}(2\pi \times 10^5) \times (1) = 100 \text{ kHz}.$

and $B_{FM} = 2(\Delta f + B) = 230 \text{ KHz}.$

$\beta = \frac{\Delta f}{B} = \frac{100}{15}$

and $B_{FM} = 2B(\beta+1) = 30\left(\frac{100}{15}+1\right) = 230 \text{ KHz}.$

For PM, :-

the peak amplitude of $\dot{m}(t)$ is 20,000 and

$$\Delta f = \frac{1}{2\pi} k_p m_p' = 50 \text{ KHz}$$

Hence

$$B_{PM} = 2(\Delta f + B) = 130 \text{ KHz}$$

Alternatively :- the deviation ratio β is

$$\beta = \frac{\Delta f}{B} = \frac{50}{15}$$

and $B_{PM} = 2B(\beta+1) = 30\left(\frac{50}{15} + 1\right) = 130 \text{ KHz}$.

(b) Doubling $m(t)$ doubles its peak value.
Hence $m_p = 2$. But B did not change $= 15$ KHz.

For FM: $\Delta f = \frac{1}{2\pi} k_f m_p = \frac{1}{2\pi}(2\pi \times 10^5)(2) = 200 \text{ KHz}$.

$$B_{FM} = 2(\Delta f + B) = 430 \text{ KHz}.$$

OR: $\beta = \frac{\Delta f}{B} = \frac{200}{15}$ and $B_{FM} = 2B(\beta+1) = 30\left(\frac{200}{15} + 1\right) = 430 \text{ KHz}$.

For PM: Doubling $m(t)$ doubles its deviation, $m_p' = 40{,}000$, and

$$\Delta f = \frac{1}{2\pi} k_p m_p' = 100 \text{ KHz}$$

$$B_{PM} = 2(\Delta f + B) = 230 \text{ KHz}$$

Alternately:- $\beta = \frac{\Delta f}{B} = \frac{100}{5}$

$$B_{PM} = 2B(\beta+1) = 30\left(\frac{100}{5} + 1\right) = 230 \text{ KHz}.$$

EX. 5.4 Repeat example 5.3 if $m(t)$ is time-expanded by a factor of 2; that is, if the period of $m(t)$ is 4×10^{-4}.

Solution :

* expanding the signal by 2 reduces the signal spectral width, bandwidth, by 2.

* $\omega_0 = 2\pi f_0 t \Rightarrow f_0 = 2.5\ KHz$.

$\therefore B = 7.5\ KHz$ (it is $nf_0 \Rightarrow 3 \times 2.5$)

$m_p = 1$, $m_p' = 10{,}000$ (m_p' of EX. 5.3 halved).

For FM : $\Delta f = \frac{1}{2\pi} k_f m_p = 100\ KHz$.

$B_{FM} = 2(\Delta f + B) = 2(100 + 7.5) = 215\ KHz$.

For PM : $\Delta f = \frac{1}{2\pi} k_p m_p' = 25\ KHz$

$B_{PM} = 2(\Delta f + B) = 65\ KHz$

EX. 55: An angle-modulated signal with carrier frequency $\omega_c = 2\pi \times 10^5$ is described by the equation

$$\phi_{EM}(t) = 10\cos(\omega_c t + 5\sin 3000t + 10\sin 2000\pi t)$$

ⓐ Find the power of the modulated signal.
ⓑ Find the frequency deviation Δf.
ⓒ Find the deviation ratio β.
ⓓ Find the phase deviation $\Delta\phi$.
ⓔ Estimate the bandwidth of $\phi_{EM}(t)$.

Solution: The signal bandwidth is the highest frequency in $m(t)$ or its deviation. In this case:

$$B = \frac{2000\pi}{2\pi} = 1000\ Hz.$$

ⓐ carrier amplitude $A = 10$, the power is $P = \frac{A^2}{2} = 50$

ⓑ Δf: $\omega_i = \frac{d}{dt}\theta(t) = \omega_c + 15{,}000\cos 3000t + 20{,}000\pi\cos 2000\pi t.$

at some phase the carrier phase components has a maximum value as $\Delta\omega = 15{,}000 + 20{,}000\pi$

$$\therefore \Delta f = \frac{\Delta\omega}{2\pi} = 12{,}387.32\ Hz.$$

ⓒ $\beta = \frac{\Delta f}{B} = \frac{12{,}387.32}{1000} = 12.387$

ⓓ The angle $\theta(t) = \omega t + (5\sin 3000t + 10\sin 2000\pi t)$. The phase deviation is the maximum value of the two sinusoidal components of $\theta(t)$.

$$\therefore \Delta\phi = 5 + 10 = 15\ rad.$$

ⓔ $B_{EM} = 2(\Delta f + B) = 26{,}774.65\ Hz.$

Some important points :-

* The instantaneous carrier frequency deviates up and down from its center frequency ω_c. This change or shift either above or below the resting or center frequency is called frequency deviation.

* The total variation in frequency from the Lowest to the highest point is called carrier swing.

$$\boxed{\text{carrier swing} = 2 \times \text{frequency deviation} = 2 \times \Delta\omega}$$

* $\Delta\omega$ depends on the amplitude (Loudness) of the message.
  * Louder sound $\Rightarrow$ Greater deviation.

* In FM broadcast, It has been internationally agreed to restrict maximum deviation to 75 KHz on each side of ω_c for sound.
  * A maximum deviation of 25 KHz for the sound portion of television broadcasts.

* Approximate FM channel width is $2 \times$ (75 KHz deviation) = 150 KHz, allowing 25 KHz guard band on either side, the channel width becomes $2(75+25) = 200$ kHz. The guardband is to prevent interference between adjacent channels.

* Notice :- In FM broadcast, the highest audio frequency transmitted is 15 KHz.

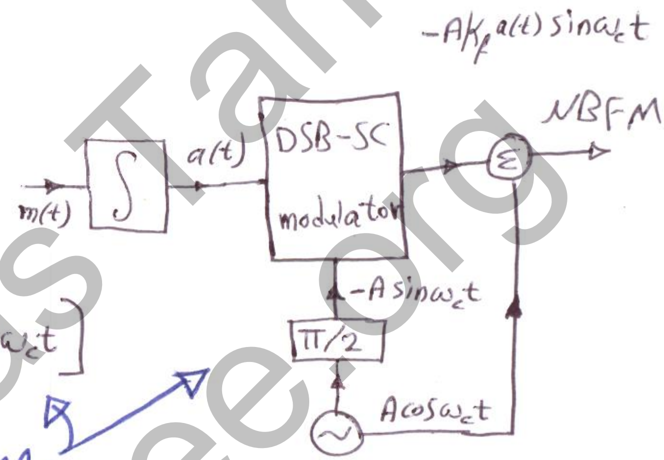
Generation of FM waves :

There are two ways :-

① Indirect generation,

② Direct generation.

① Indirect Method of Armstrong :



$$\phi_{FM}(t) \simeq A\cos\left[\omega_c t - k_f\, a(t) \sin\omega_c t\right]$$

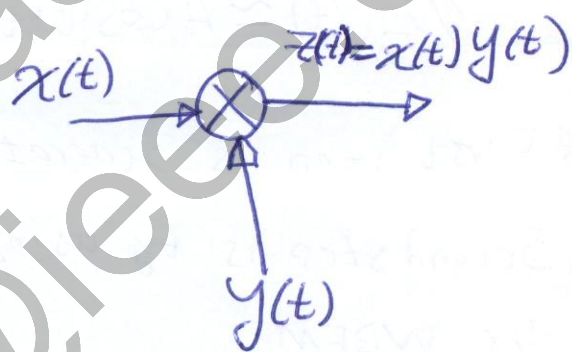
* First step is generating NBFM.
* Second step is by using the frequency multiplier to generate the WBFM.

* NBFM by indirect Armstrong method has some distortion. The distortion is due to the amplitude variation of the AM modulation (Amplitude distortion).

* Most of the distortion will be removed because of the amplitude limiter of the frequency multiplier.

Mixer OR Frequency Converter :-

* Mixer ≡ Frequency translation ≡ Frequency Converter
* Simply a mixer is a DSB-SC modulation.
* Thus, mixer is a device to convert or translate a center frequency to another center frequency.
* Let $x(t) = A\cos\omega_1 t$
  $y(t) = B\cos\omega_2 t$



then :- $Z(t) = x(t)y(t)$

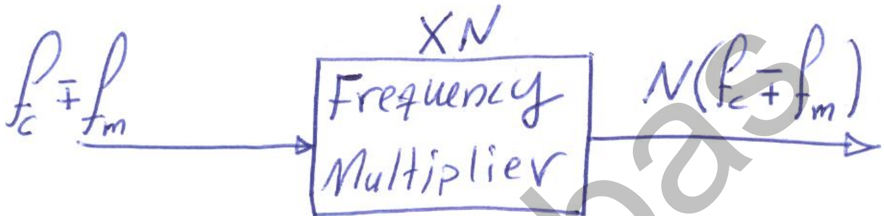
$$A\cos\omega_1 t\, B\cos\omega_2 t = \underbrace{\frac{AB}{2}\cos(\omega_1-\omega_2)t}_{\text{Down conversion}} + \underbrace{\frac{AB}{2}\cos(\omega_1+\omega_2)t}_{\text{UP conversion}}$$

* Hence :- Frequency converter or mixer or translator moves the center frequency to another Location only.

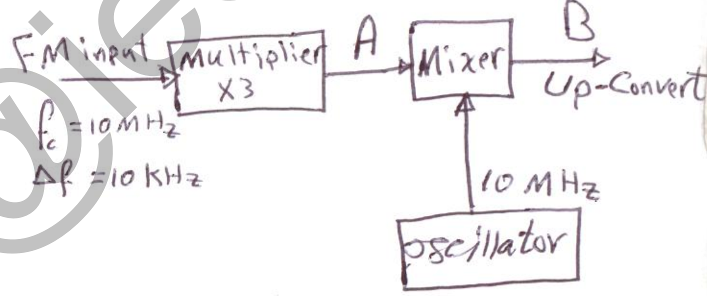
Remember this

Frequency Multiplier :

* Frequency mutiplication is the process of multiplying the frequency by a factor n.



EX. In the block diagram shown aside. Find the frequencies at points A & B.



Solution

At point (A)

$$f_{cA} = f_c \times 3 = 10 \times 3 = 30\,MHz.$$

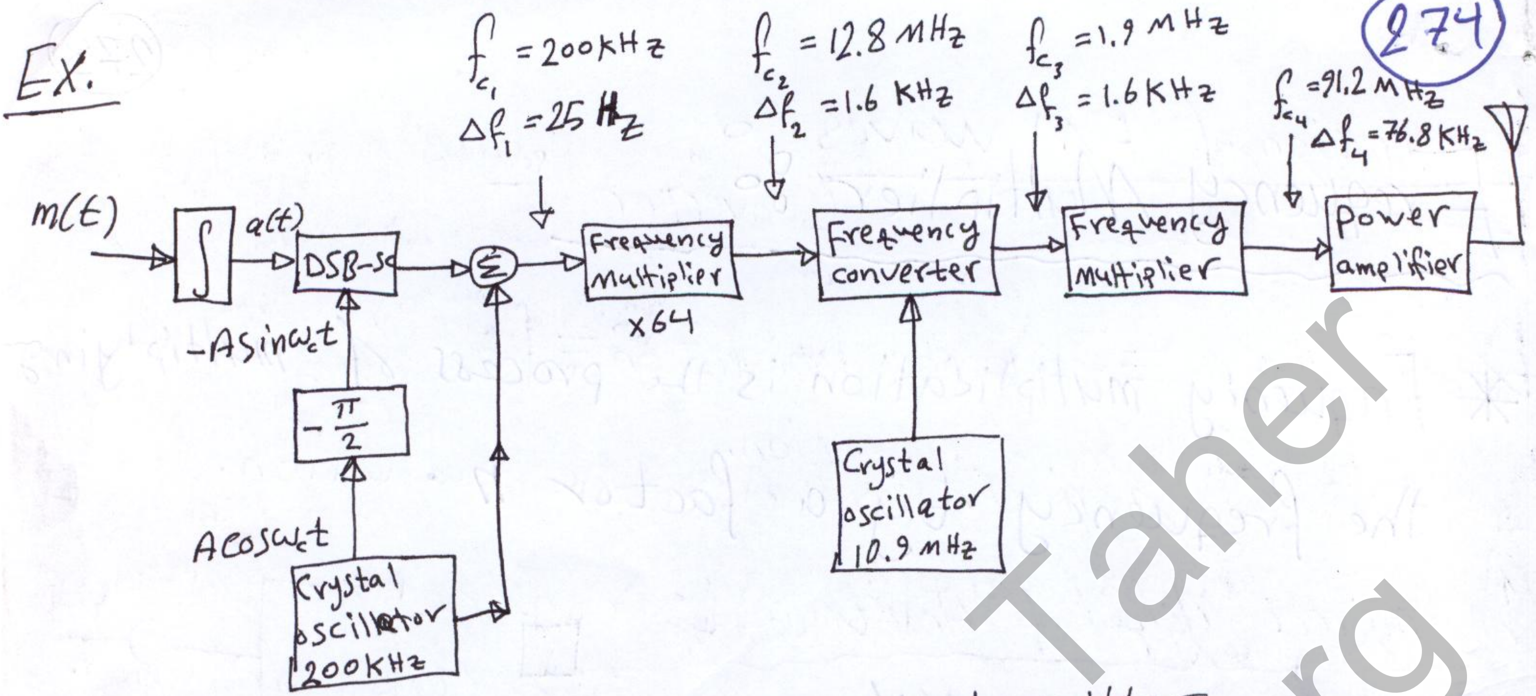
$$\Delta f_A = \Delta f \times 3 = 10k \times 3 = 30\,KHz.$$

At point (B)

$$f_{cB} = f_{cA} + 10\,MHz = 30\,MHz + 10\,MHz = 40\,MHz$$

* In other words : Frequency multiplier will multiply every thing inside the cosine or the Sine.

Ex.



Armstrong indirect FM transmitter

* Armstrong indirect method has an advantage of frequency stability, but it suffers from inherent noise caused by excessive multiplication and distortion at lower modulating frequencies, where $\Delta f / f_m$ is not small enough.

* see Example 5.6 page 230 (B. P. Lathi : Modern analog and digital communication systems).