Fundamentals of Communications Engineering

Department of Communications Engineering, College of Engineering, University of Diyala, 2016-2017

Class: Second Year
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Room: Comm-02

Lecture: 16

DSB-SC Envelope Detection

* Adding carrier to the DSC-SC at the receiver will converts the incomming signal to AM signal.

$$s(t) = \text{DSB-SC} + A\cos(2\pi f_c t + \phi)$$

$$\boxed{s(t) = x(t)\cos(2\pi f_c t) + A\cos(2\pi f_c t + \phi)}$$

$$\therefore \; S(t) = e(t)\cos[2\pi f_c t + \theta(t)] \quad \text{--- (1)}$$

where $e(t) = \sqrt{[A + x(t)]^2 - 2A\,x(t)[1 - \cos\phi]}$

and $\theta(t) = \tan^{-1}\left[\dfrac{A\sin\phi}{x(t) + A\cos\phi}\right]$

NOW $S(t) = e(t)\left[\cos(2\pi f_c t + \theta(t))\right]$

$\therefore$ The envelope is $e(t)$, if $\phi = 0$, then

$$\boxed{e(t) = A + x(t)}$$

if $\phi \neq 0$, then assuming $A \gg |x(t)|$, then

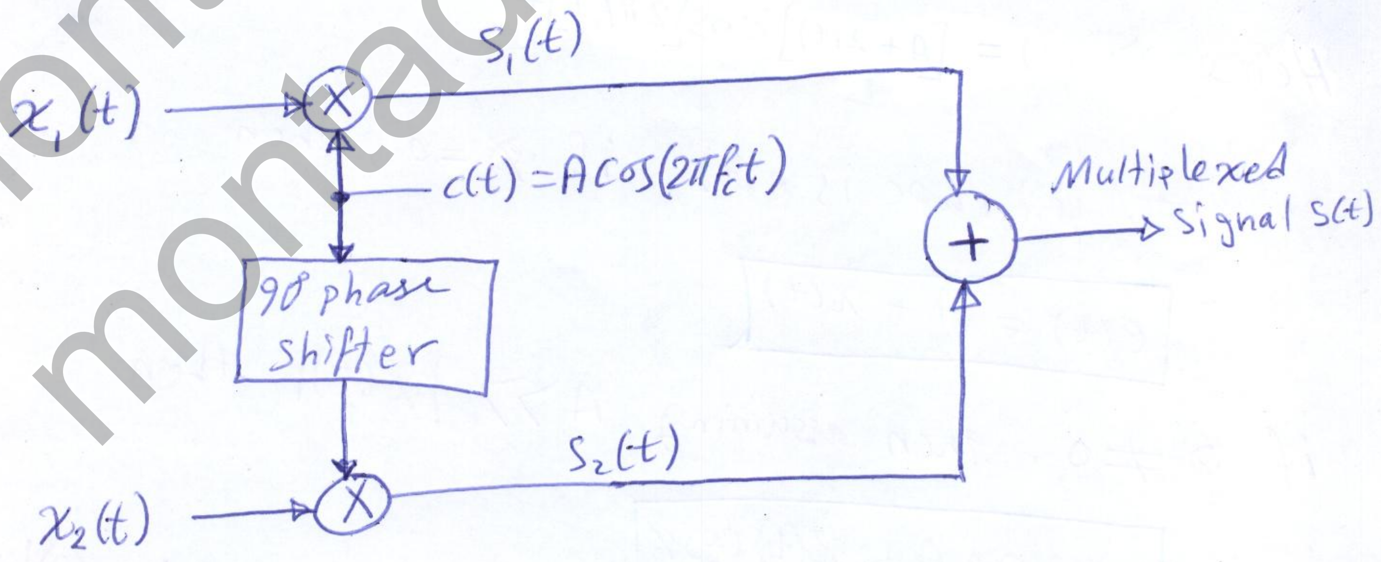
$$\boxed{e(t) \simeq A + x(t)\cos\phi}$$

* If there is frequency difference Δf and $A \gg |x(t)|$ then

$$\boxed{e(t) = A + x(t)\cos(2\pi\Delta f t)}$$

Quadrature-Amplitude Modulation (QAM)

* QAM also called Quadrature Carrier Multiplexing.
* The same frequency carrier can carry two different messages.
* QAM has the same bandwidth of AM or DSB-SC, therefore it is called bandwidth-conservation scheme.
* QAM consists of two DSB-SC signals but the carrier of the first signal is 90° phase shifted than the second carrier.



* $x_1(t)$ will modulate $c(t) = A\cos(2\pi f_c t)$

* $x_2(t)$ will modulate $c'(t) = A\sin(2\pi f_c t)$

$\therefore$ $S_1(t) = A\, x_1(t)\cos(2\pi f_c t)$

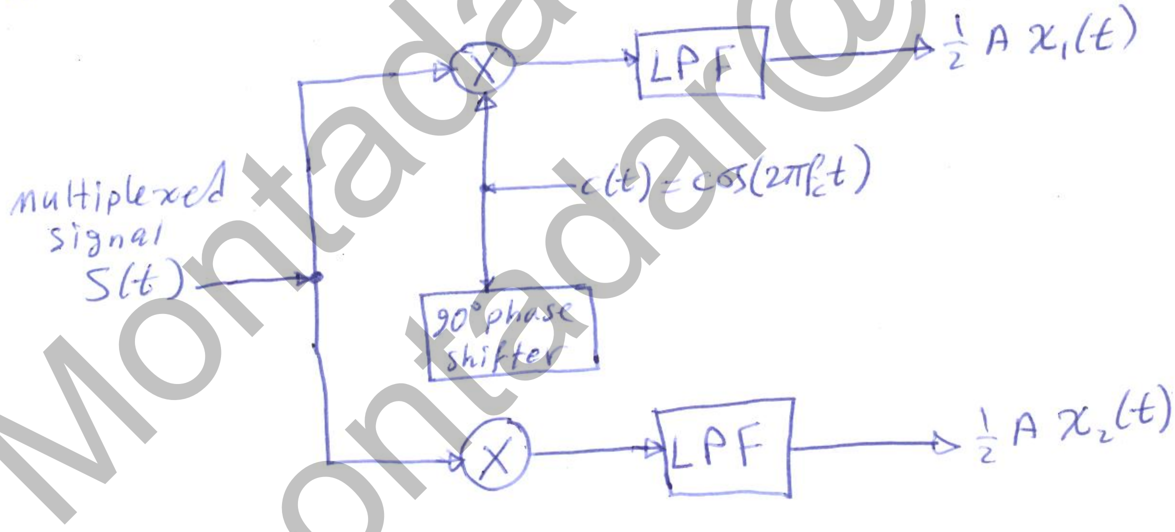
$S_2(t) = A\, x_2(t)\sin(2\pi f_c t)$

then

$S(t) = S_1(t) + S_2(t)$

$S(t) = A\left[x_1(t)\cos(2\pi f_c t) + x_2(t)\sin(2\pi f_c t)\right]$

* At the receiver



Effect of phase & frequency Errors

* In coherent detection (synchronous), the frequency and phase at the receiver must be identical to that in the transmitter.

* If either or both of F & $\emptyset$ have errors, a serious problem in the detection will happen.

* Suppose $s(t) = x(t)\cos\omega_c t$ is the transmitted signal
* Received signal is $\boxed{r(t) = [x(t)\cos\omega_c t]\cos[(\omega_c + \Delta\omega)t + \emptyset(t)]}$

$$r(t) = \frac{1}{2}x(t)\cos[(\Delta\omega)t + \emptyset] \quad -(1)$$

(i) $\Delta\omega = 0$ & $\emptyset = 0$

$r(t) = \frac{1}{2}x(t)$ no distortion

(ii) $\Delta\omega = 0$ & $\emptyset \neq 0$

$r(t) = \frac{1}{2}x(t)\cos\emptyset$ simple attenuation

but when $\emptyset = 90°$ then $r(t) =$ zero

(iii) $\Delta\omega \neq 0$ & $\emptyset = 0$

$r(t) = \frac{1}{2}x(t)\cos\Delta\omega t$ serious detection

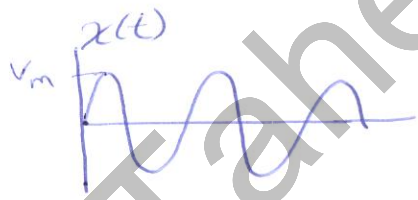
(iv) $\Delta\omega \neq 0$ & $\emptyset \neq 0$

Attenuation & distortion

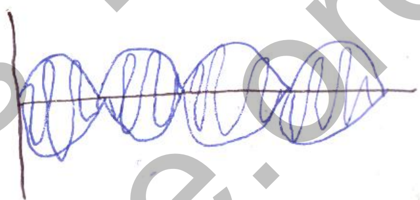
Single-Tone DSB-SC

* message signal is single-frequency sinusoidal

$$x(t) = V_m \cos(2\pi f_m t)$$



* carrier is $c(t) = A_c \cos(2\pi f_c t)$
* DSB-SC signal is $s(t) = x(t)\, c(t)$.



$$s(t) = V_m A_c \cos(2\pi f_c t) \cos(2\pi f_m t)$$

$$s(t) = \frac{V_m A_c}{2}\left[\cos 2\pi(f_c + f_m)t + \cos 2\pi(f_c - f_m)t\right]$$

$$s(t) = \frac{V_m A_c}{2}\cos\left[2\pi(f_c + f_m)t\right] + \frac{V_m A_c}{2}\cos\left[2\pi(f_c - f_m)t\right]$$

$$\therefore S(f) = \frac{V_m A_c}{4}\left[\delta(f + f_c + f_m) + \delta(f - f_c - f_m)\right] + \frac{V_m A_c}{4}\left[\delta(f + f_c - f_m) + \delta(f - f_c + f_m)\right]$$

$$S(f) = \frac{V_m A_c}{4}\left[\delta(f + f_c + f_m) + \delta(f - f_c - f_m) + \delta(f + f_c - f_m) + \delta(f - f_c + f_m)\right]$$

* There is no pure carrier

$$P_t = P_s = P_{USB} + P_{SB}$$

$$\therefore \eta = \frac{P_s}{P_t} = 100\%$$

Single Side Band-Suppressed-Carrier (SSB-SC)

* SSB-SC do not use LSB and USB to carry the same message signal.

* SSB-SC uses only one sideband, LSB or USB, to carry the message signal.

* SSB-SC bandwidth is $\frac{1}{2}$ BW of AM or DSB-SC.

* consider a message signal given as:-

$$x(t) = \cos\omega_m t \quad \text{——— (1)}$$

* Double sideband modulation is multiplying (1) by c(t)

$$c(t) = \cos\omega_c t \quad \text{——— (2)}$$

then

$$s(t) = x(t)\, c(t) \quad \text{——— (3)}$$

$$s(t) = \cos\omega_m t \cos\omega_c t$$

$$s(t) = \frac{1}{2}\left[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t\right] \quad \text{——— (4)}$$

$\therefore$ $$s(t)_{LSB} = \frac{1}{2}\cos(\omega_c-\omega_m)t \quad \text{——— (5)}$$

$$s(t)_{USB} = \frac{1}{2}\cos(\omega_c+\omega_m)t \quad \text{——— (6)}$$

* The LSB signal part $s(t)_{LSB} = \cos[(\omega_c - \omega_m)t]$:

$$\cos(\omega_c - \omega_m)t = \cos\omega_m t \cos\omega_c t + \sin\omega_m t \sin\omega_c t$$

* The USB signal part $s(t)_{USB} = \cos[(\omega_c + \omega_m)t]$:

$$\cos(\omega_c + \omega_m)t = \cos\omega_m t \cos\omega_c t - \sin\omega_m t \sin\omega_c t$$

* combining both sides

$$\boxed{s(t)_{SSB} = \cos\omega_m t \cos\omega_c t \pm \sin\omega_m t \sin\omega_c t} \quad (7)$$

* In $S(t)_{SSB}$, the + sign represents the lower sideband,
the − sign represents the upper sideband.

Hence : $S(t)_{SSB} = \cos\omega_m t \cos\omega_c t + \underbrace{\sin\omega_m t}_{\cos(\omega_m t - \frac{\pi}{2})} \sin\omega_c t - \underbrace{\sin\omega_m t}_{\cos(\omega_m t - \frac{\pi}{2})} \sin\omega_c t$

~~where~~ $\cancel{\sin\omega_c t = \cos(\omega_c t - \frac{\pi}{2})}$

$$\sin\omega_m t = \cos\left(\omega_m t - \frac{\pi}{2}\right)$$

Thus $S(t)_{SSB} = \underbrace{\cos\omega_m t}_{x(t)} \cos\omega_c t + \underbrace{\cos\left(\omega_m t - \frac{\pi}{2}\right)}_{x_h(t)} \sin\omega_c t$
$- \cos\left(\omega_m t - \frac{\pi}{2}\right) \sin\omega_c t$

OR

$$\boxed{S(t)_{SSB} = x(t)\cos\omega_c t \pm x_h(t)\sin\omega_c t} \quad (8)$$

HiLBert Transform

* A $\left(-\frac{\pi}{2}\right)$ phase shif to every frequency component of a signal is called Hilbert-Transform.

* If a function is given by $g(t)$ then its Hilbert transform is $g_h(t)$.

$$\boxed{g(t) \xleftrightarrow{H.T} g_h(t)}$$

$$\boxed{g_h(t) = \frac{1}{\pi} g(t) * \frac{1}{t} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{g(\tau)}{t-\tau}\, d\tau}$$

Inverse Hilbert Transform is

$$\boxed{g(t) = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{g_h(\tau)}{t-\tau}\, d\tau}$$

Example Find the Hilbert transform of a signal $x(t) = \cos\omega t$.

Solution $x(t) = \cos\omega t$

$$x_h(t) = \frac{-1}{\pi}\int_{-\infty}^{\infty}\frac{\cos\omega\tau}{\tau - t}\,d\tau = \frac{-1}{\pi}\int_{-\infty}^{\infty}\frac{\cos[\omega(y-t)]}{y}\,dy$$

$$x_h(t) = \frac{-1}{\pi}\left\{\cos\omega t\cancelto{0}{\int_{-\infty}^{\infty}\frac{\cos\omega y}{y}\,dy} - \sin\omega t\cancelto{\pi}{\int_{-\infty}^{\infty}\frac{\sin\omega y}{y}\,dy}\right\}$$

$$= \frac{-1}{\pi}\left[-\pi\sin\omega t\right]$$

$$\therefore\ x_h(t) = \sin\omega t$$

* Thus Hilbert transform is a delay of $\frac{\pi}{2}$ at all frequencies.

* $e^{j2\pi f_0 t} \overset{H.T}{\longleftrightarrow} e^{j2\pi f_0 t - \frac{\pi}{2}} = -j\, e^{j2\pi f_0 t}$

* $e^{-j2\pi f_0 t} \overset{H.T}{\longleftrightarrow} e^{-j(2\pi f_0 t - \frac{\pi}{2})} = j\, e^{j2\pi f_0 t}$

In other words:

* $f > 0$, the spectrum multiplied by $-j$
* $f < 0$, the spectrum multiplied by $+j$

Hence the frequency spectrum of the signal is multiplied by $-j\,\mathrm{sgn}(f)$

$$\boxed{X_h(f) = -j\,\mathrm{sgn}(f)\,X(f)}$$ Hilbert transform in the Frequency Domain.

* Since HT changes cosines into sines, HT $x_h(t)$ of a signal $x(t)$ is orthogonal to $x(t)$.
* Applying HT two times produces 180° phase shif, i.e., it is a sign reversal of the original signal.

Hilbert Transform Properties

1. If $x(t)$ even $\xleftrightarrow{F.T.}$ $X(f)$ real & even

   therefore : $-j\,sgn(f)\,X(f)$ imaginary & odd

   Hence : $F.T.^{-1}[X_h(f)]$ is odd.

2. If $x(t)$ odd $\xleftrightarrow{F.T.}$ $X(f)$ imaginary & odd

   therefore : $-j\,sgn(f)\,X(f)$ real & even

   Hence : $F.T.^{-1}[X_h(f)]$ is even

3. $x(t) \xleftrightarrow{H.T.} x_h(t) \xleftrightarrow{H.T.} -x(t)$

4. The energy of $x(t)$ = energy of $x_h(t)$.

5. $x(t) \perp x_h(t)$

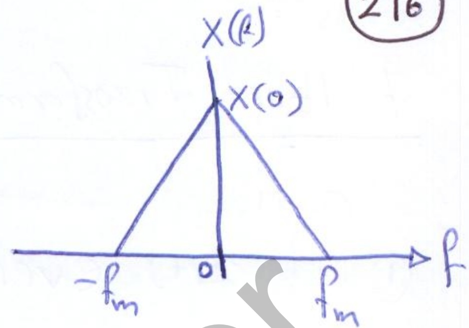
Pre-Envelope or Analytical Signal

a signal $x(t) \xleftrightarrow{H.T} x_h(t)$

pre_Envelope of $x(t)$ is $x_p(t)$

$$\boxed{x_p(t) = x(t) + j\,x_h(t)} \qquad \boxed{x_p^*(t) = x(t) - j\,x_h(t)}$$

* $x_p(t)$ is very useful in SSB derivation.

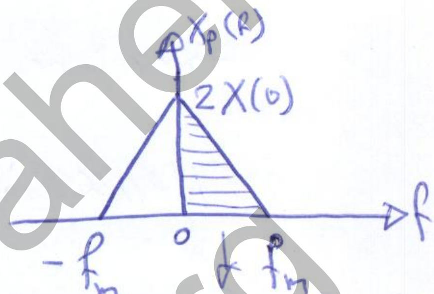


* The pre-envelope of $x(t)$ is

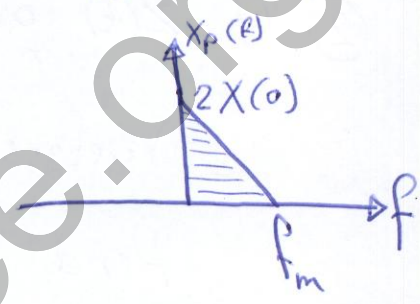
$$x_p(t) = x(t) + j\,x_h(t)$$

$$X_p(f) = X(f) + j\left[-j\,\mathrm{sgn}(f)\,X(f)\right]$$

$$X_p(f) = X(f) + X(f)\,\mathrm{sgn}(f)$$



since $\mathrm{sgn}(f) = \begin{cases} 1 & \text{for } f>0 \\ -1 & \text{for } f<0 \end{cases}$



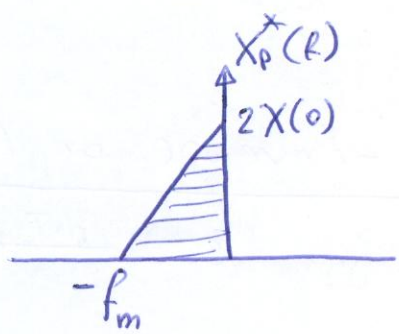
$\therefore$ $X_p(f) = X(f) + X(f)(+1)$

$X_p(f) = 2X(f)$ for $f>0$

$X_p(f) = 0$ for $f<0$

NOW: $X_p^*(f) = X(f) - j\left[-j\,X(f)\,\mathrm{sgn}(f)\right]$

$X_p^*(f) = X(f) - X(f)\,\mathrm{sgn}(f)$

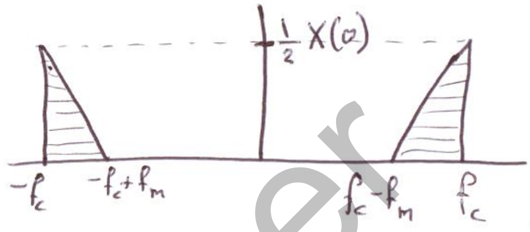


$X_p^*(f) = 0$ For $f>0$

$X_p^*(f) = 2X(f)$ for $f<0$

* For a general signal $x(t)$, the lower sideband modulated signal is

$$s(t)_{LSSB} = \frac{1}{4}\left[x_p^*(t)\,e^{j\omega_c t} + x_p(t)\,e^{-j\omega_c t}\right]$$



LSB-SSB-SC

$$s(t)_{LSSB} = \frac{1}{2}\left[x(t) - j\,x_p(t)\right]e^{j\omega_c t} + \frac{1}{2}\left[x(t) + j\,x_h(t)\right]e^{-j\omega_c t}$$

$$= \frac{1}{2}x(t)\left[\frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}\right] + \frac{j}{2}x_h(t)\left[\frac{e^{-j\omega_c t} - e^{j\omega_c t}}{2}\right]$$

$$s(t)_{LSSB} = \frac{1}{2}\left[x(t)\cos\omega_c t + x_h(t)\sin\omega_c t\right]$$

* For a general signal $x(t)$, similarly, the upper sideband modulated signal is

$$s(t)_{USSB} = \frac{1}{2}\left[x(t)\cos\omega_c t - x_h(t)\sin\omega_c t\right]$$

* Thus,

$$\boxed{s(t)_{SSB} = x(t)\cos\omega_c t \pm x_h(t)\sin\omega_c t}$$

+ sign represents Lower sideband (LSB-SSB-SC)

− sign represents Upper sideband (USB-SSB-SC)

SSB Modulation Advantages :

① Less bandwidth, therefore more message signals can be transmitted on one frequency carrier.

② power saving, at 100% modulation, power saving is 83.3%

③ Reduced interference noise, this is due to reduced bandwidth.

SSB Modulation Disadvantages :

① Generation and reception of SSB are very complicated.

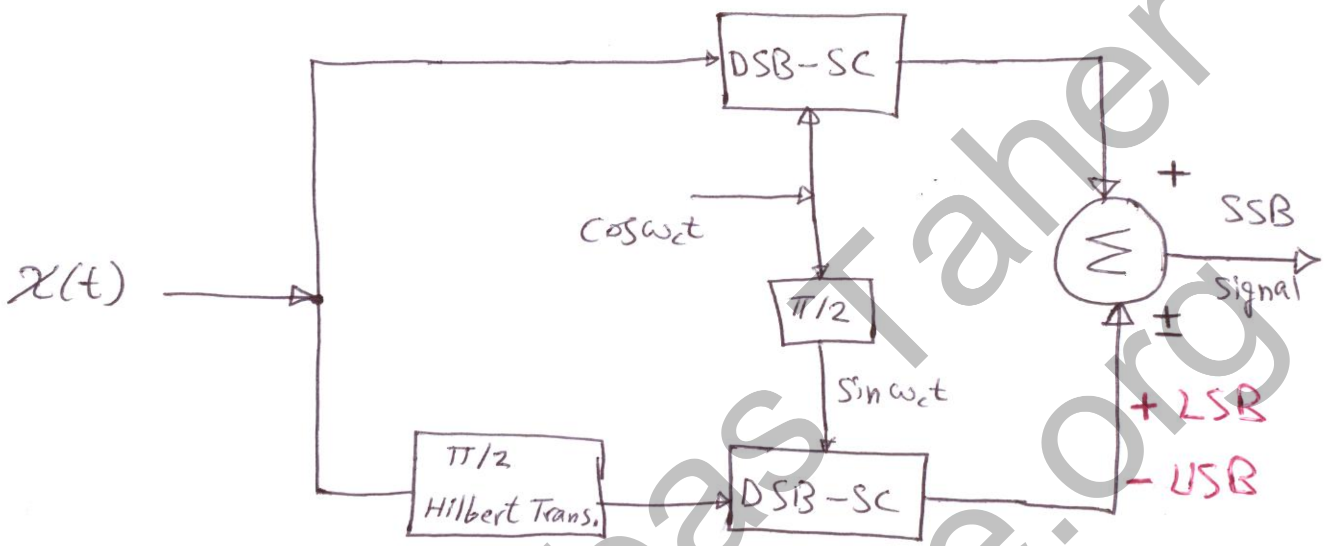
② T_x & R_x SSB systems need very excellent frequency stability.

* A slight change in frequency will distort the quality of transmitted and received signal.

* Therefore, SSB is not generally used for the transmission of good quality music.

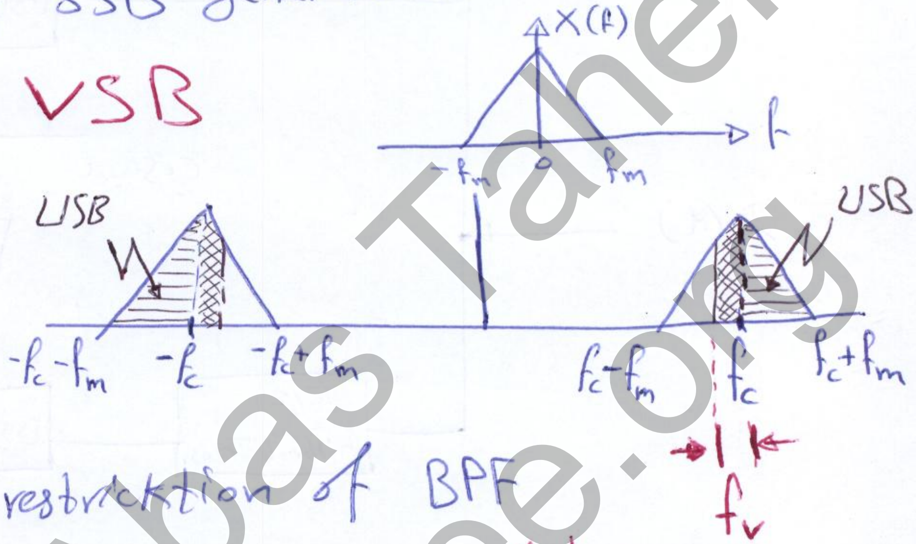
* It is used for speech transmission.

SSB Modulation Generation

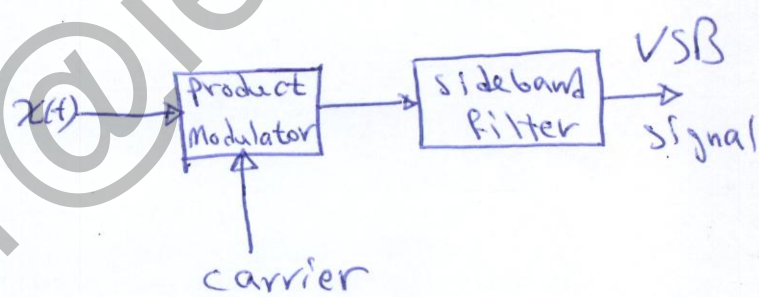


Vestigial Sideband Modulation (VSB)

* The complexity of SSB generation motivates the introduction of VSB



* VSB relaxed the restriction of BPF by allowing part of LSB to be passed by the BPF.



* $BW_{VSB} > B_{SSB}$

∴ $\boxed{B_{VSB} = (f_m + f_v)}$ Hz

* VSB has become standard for TV signal transmission