Fundamentals of Communications Engineering

Department of Communications Engineering, College of Engineering, University of Diyala, 2016-2017

Class: Second Year
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Room: Comm-02

Lecture: 15

Multiple-Tone AM :

* in multiple tone AM, the message signal $x(t)$ consists of multiple frequencies as:

$$x(t) = V_1 \cos(2\pi f_{m1} t) + V_2 \cos(2\pi f_{m2} t) + V_3 \cos(2\pi f_{m3} t) + \cdots$$

since $s(t) = A \cos(2\pi f_c t) + x(t) \cos(2\pi f_c t)$

$$\therefore \quad s(t) = A \cos(2\pi f_c t) + \cos(2\pi f_c t)\left[V_1 \cos(2\pi f_{m1} t) + V_2 \cos(2\pi f_{m2} t) + V_3 \cos(2\pi f_{m3} t) + \cdots\right]$$

or $$s(t) = A\left[1 + \frac{V_1}{A} \cos(2\pi f_{m1} t) + \frac{V_2}{A} \cos(2\pi f_{m2} t) + \frac{V_3}{A} \cos(2\pi f_{m3} t) + \cdots\right]$$

since $\mu_a = \frac{V_m}{A}$

$$\therefore \quad s(t) = A\left[1 + \mu_{a1} \cos(2\pi f_{m1} t) + \mu_{a2} \cos(2\pi f_{m2} t) + \mu_{a3} \cos(2\pi f_{m3} t) + \cdots\right]$$

Total Power : $\boxed{P_t = P_c + P_s}$

$$P_s = \frac{1}{2}\left[\frac{(\mu_{a1} A)^2}{2} + \frac{(\mu_{a2} A)^2}{2} + \cdots\right] = \frac{1}{4} A^2 \left[\mu_{a1}^2 + \mu_{a2}^2 + \mu_{a3}^2 + \cdots\right]$$

$$\boxed{P_t = P_c \left[1 + \frac{\mu_{a1}^2}{2} + \frac{\mu_{a2}^2}{2} + \frac{\mu_{a3}^2}{2} + \cdots\right]}$$

Total Modulation Index : $\boxed{\mu_{at} = \sqrt{\mu_{a1}^2 + \mu_{a2}^2 + \cdots}}$

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The power for AM wave is also expressed as

$$P_t = P_c\left(1+\frac{m_t^2}{2}\right) \quad ...(2.47)$$

Comparing equations (2.46) and (2.47), we get

$$m_t^2 = m_1^2 + m_2^2 + m_3^2 + ...+ m_n^2$$

or $$m_t = \sqrt{m_1^2 + m_2^2 + m_3^2 + ...+ m_n^2} \quad ...(2.48)$$

This is the desired expression for the total or net modulation index.*

EXAMPLE 2.12. An AM transmitter radiates 9 kW of power when the carrier is unmodulated and 10.125 kW when the carrier is sinusoidally modulated. Find the modulation index, percentage of modulation. Now, if another sine wave, corresponding to 40 percent modulation is transmitted simultaneously, then calculate the total radiated power. (Very Important)

Solution : (i) We know that for a single-tone sinusoidal amplitude-modulation the total power is expressed as

$$P_t = P_c\left(1+\frac{m_a^2}{2}\right) \quad ...(i)$$

where P_t = modulated or total power

P_c = unmodulated or carrier power

m_a = modulation index

Given that, $P_t = 10.125$ kW

and $P_c = 9$ kW

Using equation (i), we get

$$1+\frac{m_a^2}{2} = \frac{P_t}{P_c}$$

or $$\frac{m_a^2}{2} = \frac{P_t}{P_c} - 1 = \frac{10.125}{9} - 1 = 1.125 - 1 = 0.125$$

or $$m_a^2 = 0.125 \times 2 = 0.250 = 0.50$$

(ii) We know that in case of modulation by two sinusoidal waves, the total modulation index m_t is expressed as

$$m_t = \sqrt{m_1^2 + m_2^2}$$

Let $m_1 = m_a = 0.5$

Given that $m_2 = 0.4$

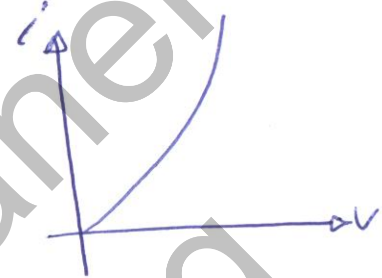
Therefore, $$m_t = \sqrt{(0.5)^2 + (0.4)^2} = \sqrt{0.25 + 0.16} = \sqrt{0.41} = 0.64$$

The total radiated power in this case will be

$$P_t = P_c\left(1+\frac{m_t^2}{2}\right) = 9 \times \left(1+\frac{0.64^2}{2}\right) = 9\,(1 + 0.205) = 10.84 \text{ kW}$$ **Ans.**

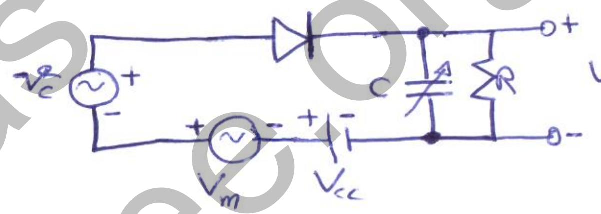
AM Generation Using Square Law

* square law diode is a normal diode but we will make use of its non-linear region.



* Assume carrier voltage is $v_c = V_c \cos\omega_c t$
* Assume the modulating voltage is $v_m = V_m \cos\omega_m t$

The ac voltage across the diode is



$$v_s = v_c + v_m$$

$$v_s = V_c \cos\omega_c t + V_m \cos\omega_m t \quad \text{— (1)}$$

The non Linear relationship of the diode is

$$i = a + b v_s + c v_s^2 \quad \text{— (2)} \qquad [a, b, \text{ and } c : \text{constants}]$$

$$i = a + b(V_c \cos\omega_c t + V_m \cos\omega_m t) + c(V_c \cos\omega_c t + V_m \cos\omega_m t)^2$$

$$i = \underbrace{\left(a + \tfrac{1}{2} c V_c^2 + \tfrac{1}{2} c V_m^2\right)}_{1} + \underbrace{b V_c \cos\omega_c t}_{2} + \underbrace{b V_m \cos\omega_m t}_{3}$$

$$+ \underbrace{\left(\tfrac{1}{2} c V_c^2 \cos 2\omega_c t + \tfrac{1}{2} c V_m^2 \cos 2\omega_m t\right)}_{4} + \underbrace{c V_c V_m \cos(\omega_c + \omega_m)t}_{5}$$

$$+ \underbrace{c V_c V_m \cos(\omega_c - \omega_m)t}_{6}$$

1 DC 2 carrier 3 modulating signal 4 harmonics

5 USB 6 LSB

* The tuned circuit is a BPF, then only ω_c, $\omega_c+\omega_m$ & $\omega_c-\omega_m$ will pass and the other components will be rejected.

$$\therefore\ i = bV_c\cos\omega_c t + cV_cV_m\cos(\omega_c+\omega_m)t + cV_cV_m\cos(\omega_c-\omega_m)t$$

$$= bV_c\cos\omega_c t + cV_cV_m\left[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t\right]$$

$$= bV_c\cos\omega_c t + 2cV_cV_m\cos\omega_c t\cos\omega_m t$$

$$= bV_c\left[1+\frac{2cV_m}{b}\cos\omega_m t\right]\cos\omega_c t$$

$$= bV_c\left(1+\mu_a\cos\omega_m\right)\cos\omega_c t$$

$\therefore$ modulation index $\mu_a = m_a = \dfrac{2cV_m}{b}$

AM Demodulation

* Two main classes of AM demodulations are:-

① Square-Law Detector

② Envelope Detector

Square-Law Detector

* To use square-Law detector, the non-linear region of the diode can be used.

* To use the non-Linear region of the diode, the input magnitude must be small (Less than 1 Volts)

$$i = aV + bV^2 \quad \text{—— (1)}$$

where V is the modulated input signal

$$V = A[1 + m_a \cos\omega_m t] \cos\omega_c t \quad \text{—— (2)}$$

$$i = a[A(1 + m_a \cos\omega_m t)\cos\omega_c t] + b[A(1 + m_a \cos\omega_m t)\cos\omega_c t]^2$$

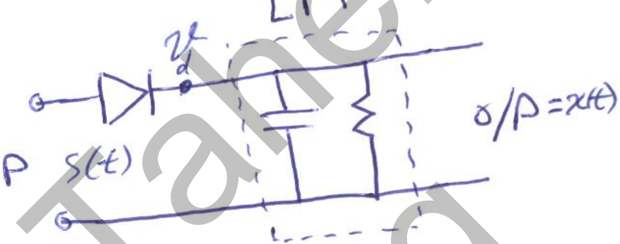
* we get many terms as

1. DC
2. $\cos \omega_c t$ carrier
3. $\cos \omega_m t$ message
4. $\cos 2\omega_c t$ harmonic of carrier
5. $\cos 2\omega_m t$ harmonic of message
6. $\cos(\omega_c+\omega_m)t$ USB
7. $\cos(\omega_c-\omega_m)t$ LSB
8. $\cos(2\omega_c+\omega_m)t$ Double carrier USB
9. $\cos(2\omega_c-2\omega_m)t$ Double carrier 2LSB
10. $\cos(2\omega_c-\omega_m)t$ Double carrier LSB
11. $\cos(2\omega_c+2\omega_m)t$ Double carrier 2USB

* LPF will pass only third term (message).

Envelope Detector

* Envelope detector also called Linear detector because it is a Linear diode.



* Envelope detector consists of :-

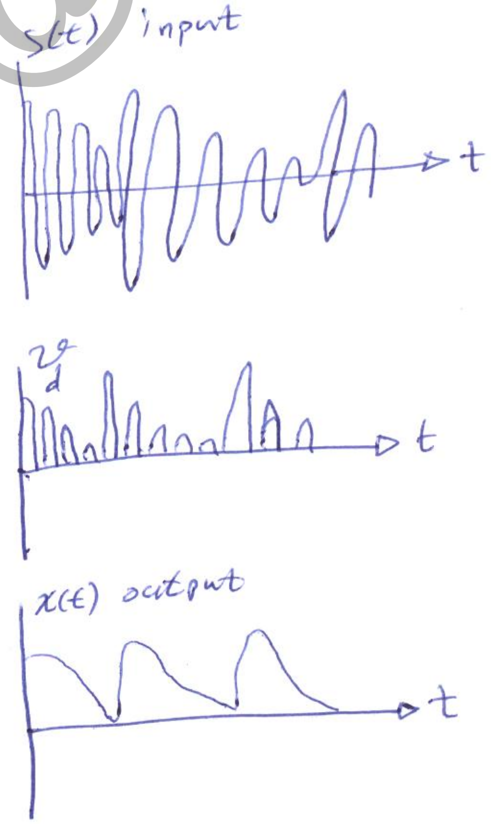
  ① Half-wave rectifier

  ② Low-pass-Filter

* Half-wave rectifier cut-off the Lower amplitude.

* The LPF, which has cut-off frequency of B, where B is the bandwidth of the message signal

$$\boxed{B = f_m}$$



AM modulation Summary

1. AM also called Double side band-Full carrier,
2. Bandwidth requirement $= 2f_m$,
3. transmitter is less complexity,
4. Receiver is easy detector,
5. AM receivers are cost efficient,
6. AM waves can travel longer distances,
7. AM used in Radio Broadcasting, and
8. AM used in Picture transmission of TV systems.

Double Side Band Suppressed Carrier (DSB-SC)

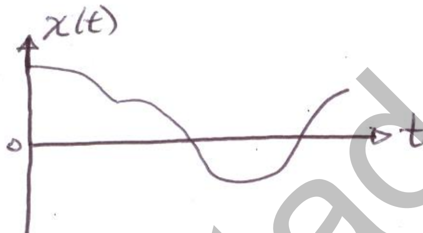
* Assume the message signal is $x(t) \xleftrightarrow{F.T.} X(f)$

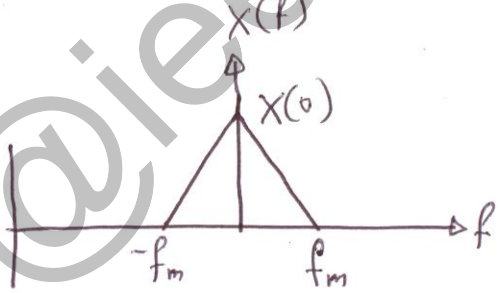
* Let the carrier signal $c(t) = A_c \cos\omega_c t \xleftrightarrow{F.T.} \frac{A_c}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right]$

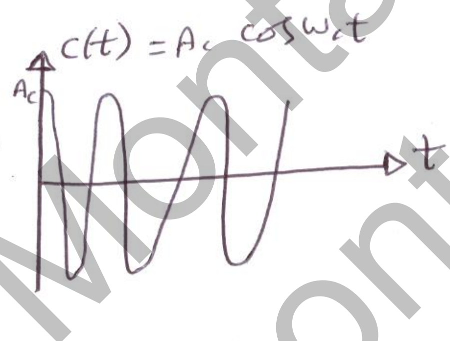
$\therefore$ DSB-SC is $A_c x(t)\cos\omega_c t = s(t)$

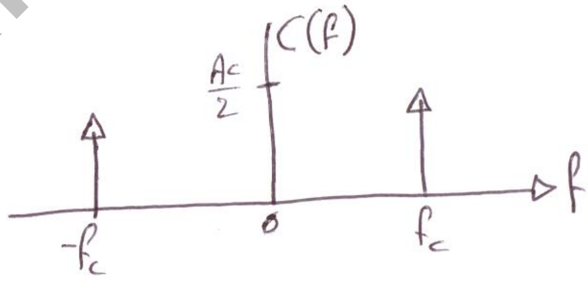
$\therefore$ DSB-SC does not has the term of the pure carrier

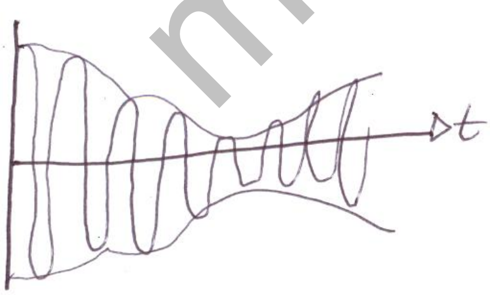
$$s(t) = A_c x(t)\cos\omega_c t \xleftrightarrow{F.T.} S(f) = \frac{A_c}{2}\left[X(f-f_c)+X(f+f_c)\right]$$

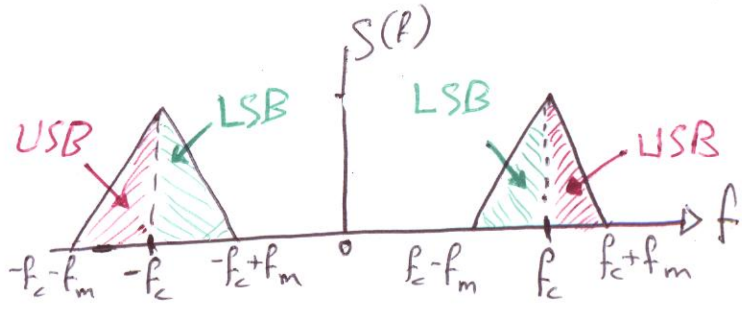








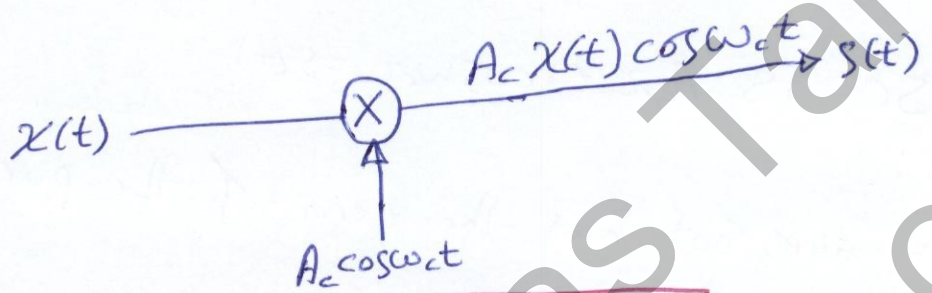




$$BW_{DSB\text{-}SC} = B = 2f_m$$

DSB-SC signal Generation

$s(t) = A_c x(t) \cos\omega_c t$ is simply a carrier $A_c \cos\omega_c t$ multiplied by the message signal $x(t)$.

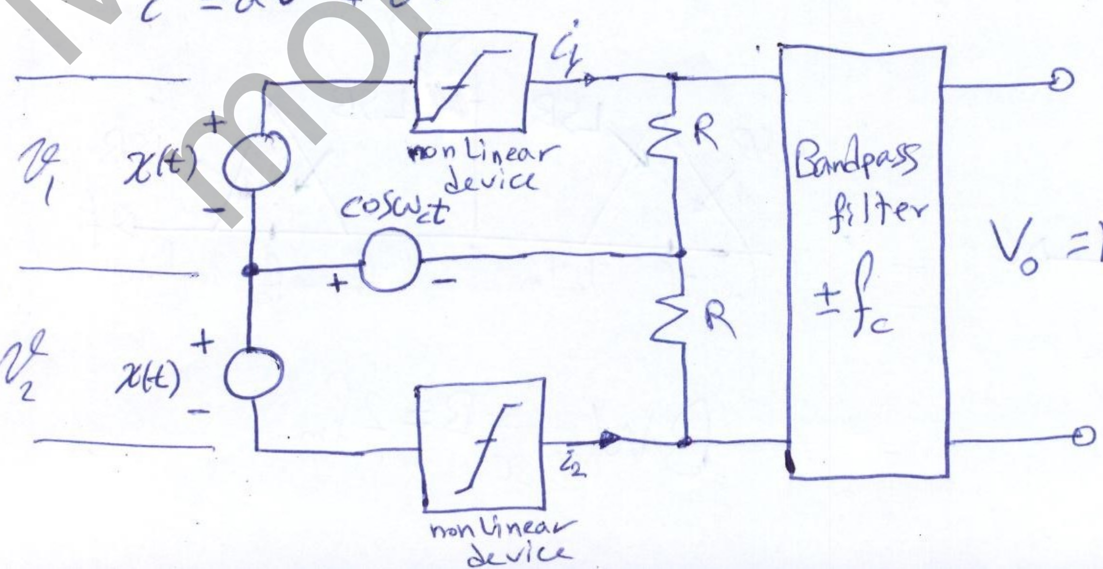


$\boxed{\text{product Modulator}}$

* Another type of DSB-SC modulator is by using Non-linear Devices or Balanced Modulator.

* Non-linear devices employed in Balanced modulator is diode.

$i = av + bv^2$ a & b constants

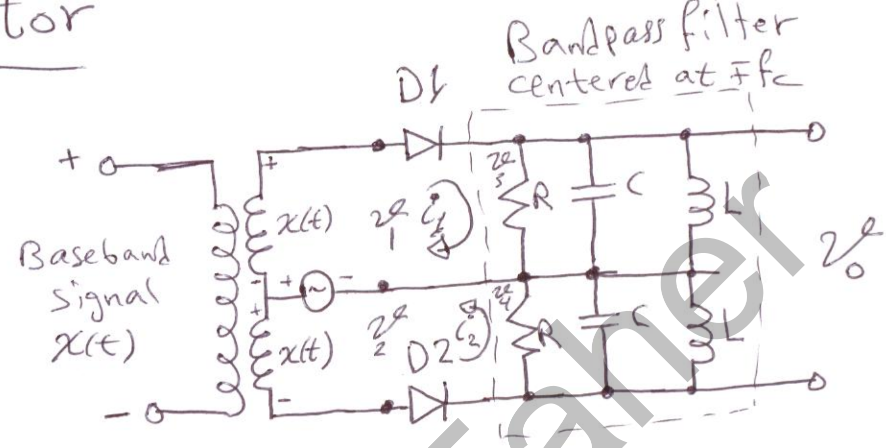


$V_o = K\, x(t) \cos\omega_c t$

Balanced Modulator

* The nonlinear v-i relationship of a diode is given as

$$i = av + bv^2 \qquad (1)$$



* v_1 & v_2 are

$$v_1 = \cos\omega_c t + x(t) \qquad (2)$$

$$v_2 = \cos\omega_c t - x(t) \qquad (3)$$

* For the first diode D1 :

$$i_1 = a v_1 + b v_1^2 \qquad (4)$$

* For the second diode D2 :

$$i_2 = a v_2 + b v_2^2 \qquad (5)$$

Hence: $i_1 = a[\cos\omega_c t + x(t)] + b[\cos\omega_c t + x(t)]^2$

$$\therefore i_1 = a\cos\omega_c t + a x(t) + b\cos^2\omega_c t + b x^2(t) + 2b\, x(t)\cos\omega_c t \qquad (6)$$

and similarly

$$i_2 = a\cos\omega_c t - a x(t) + b\cos^2\omega_c t + b x^2(t) - 2b\, x(t)\cos\omega_c t \qquad (7)$$

$$v_o = v_3 - v_4 \qquad (8)$$

$$v_o = R i_1 - R i_2 = R(i_1 - i_2) \qquad (9)$$

$$\therefore v_o = [2a x(t) + 4b\, x(t)\cos\omega_c t]R \qquad (10)$$

since v_o is accross BPF centered at $\mp\omega_c$

$$\therefore v_o = 4Rb\, x(t)\cos\omega_c t$$

$$\boxed{v_o = A\, x(t)\cos\omega_c t}$$

DSB-SC Demodulation

* DSB-SC can be demodulated in two methods :-
  1. synchronous detector, and
  2. Envelope detector.

* In general, DSB-SC demodulation is by multiplying the modulated signal by the carrier itself.

$S(t) = A_c\, x(t) \cos\omega_c t$ —— (1) modulated signal

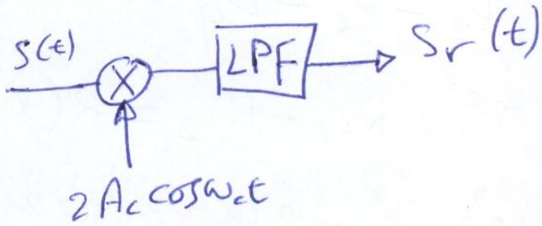
$c(t) = A_c \cos\omega_c t$ —— (2) carrier signal

$x(t)$ is the message signal.

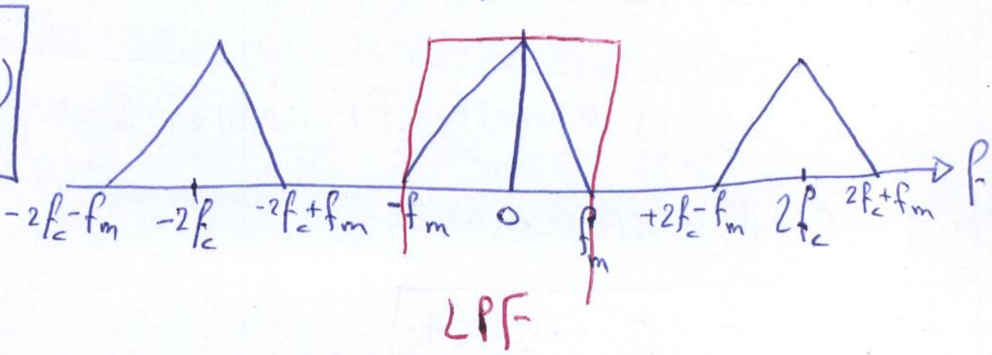
$$S(t)_r = \left[A_c\, x(t) \cos\omega_c t\right] \cdot 2A_c \cos\omega_c t$$

$$= 2A_c^2\, x(t) \cos^2\omega_c t = A_c^2\, x(t)\left[1 + \cos 2\omega_c t\right]$$

$$= A_c^2\, x(t) + \underbrace{A_c^2\, x(t) \cos 2\omega_c t}_{\text{filtered out by LPF}}$$

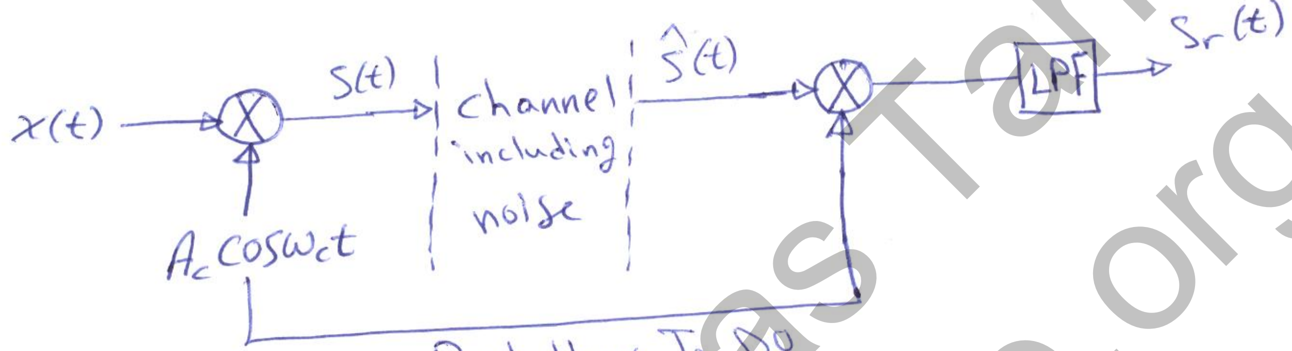


$$\boxed{S(t)_r = A_c^2\, x(t)}$$



① Synchronous Detection

This type of demodulation stands for a detection by using the carrier of the transmitter at the receiver.



But How To Do This?

* The synchronous detector requires two conditions :-

① the carrier should be provided at the receiver without error.

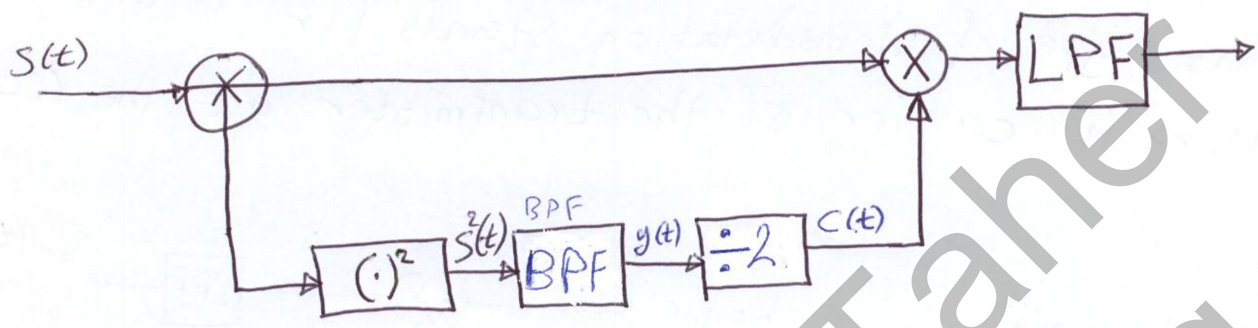
② the phase of the carrier at the receiver must be same as the phase at the transmitter.

* The synchronized carrier can be provided at the receiver by two methods:-

(i) square-Law detector.

(ii) phase-Locked Loop.

squaring-Law detection :-



$s(t) = A_c\, x(t) \cos\omega_c t$

$s^2(t) = A_c^2\, x^2(t) \cos^2\omega_c t = A_c^2\, x^2(t) \left[\frac{1}{2} + \frac{1}{2}\cos 2\omega_c t\right]$

$s^2(t) = A_c^2\, x^2(t)\, \frac{1}{2} + \frac{1}{2} A_c^2\, x^2(t) \cos 2\omega_c t$

* After BPF

$y(t) = \frac{1}{2} A_c^2\, x^2(t) \cos 2\omega_c t$

* After frequency division by 2

$\boxed{c(t) = \frac{1}{2} A_c^2\, x^2(t) \cos\omega_c t}$

OR

$\boxed{c(t) = K \cos\omega_c t}$

Phase-Locked Loop detector [PLL]

we will consider it Later