Fundamentals of Communications Engineering

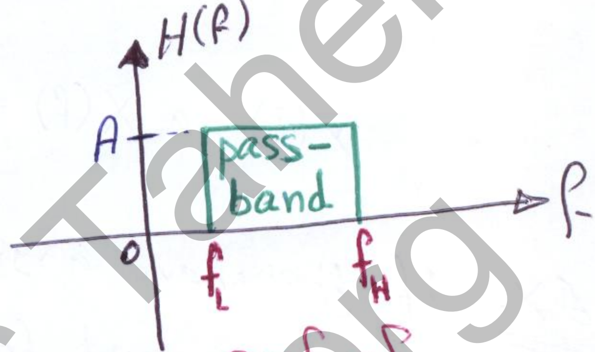
Department of Communications Engineering, College of Engineering, University of Diyala, 2016-2017

Class: Second Year
Instructor: Dr. Montadar Abas Taher
Room: Comm-02

Lecture: 13

Filters :-

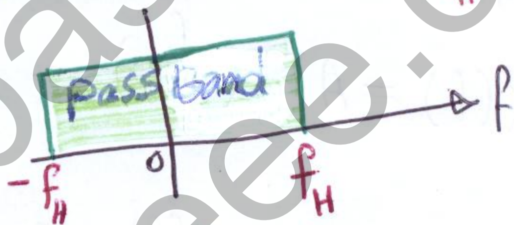
The filter is an electronic circuit, which passes interested frequency band and rejects the unwanted frequency, which is out of the interested band.



$B = f_H - f_L$

* if $f_L = 0$ & $f_H \neq \infty$ (finite)

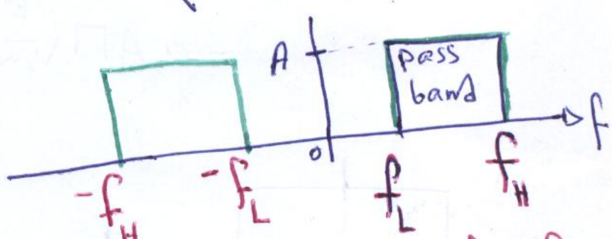
then we have Low-pass-filter (LPF)



$B_{LPF} = f_H$

* if $f_L > 0$ & $f_H \neq \infty$ (finite)

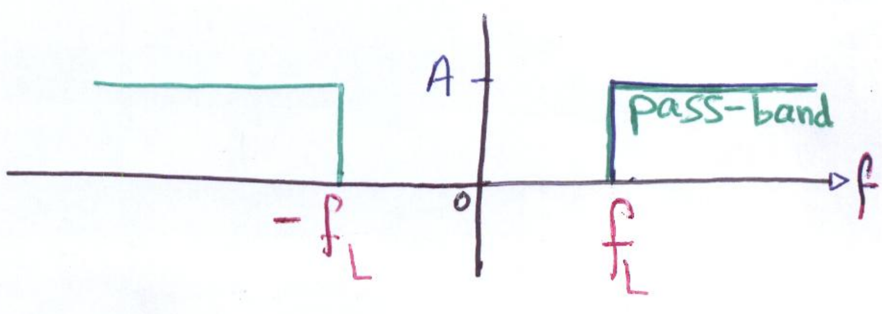
then we have Band-Pass-Filter (BPF)



$B_{BPF} = f_H - f_L$

* if $f_L > 0$ & $f_H = \infty$

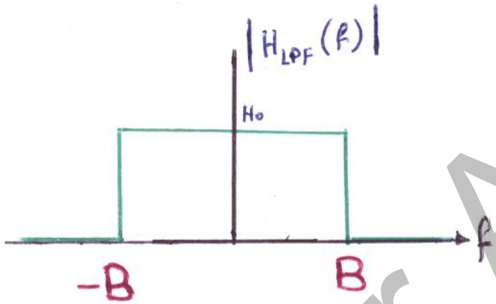
then we have High-pass-filter (HPF)

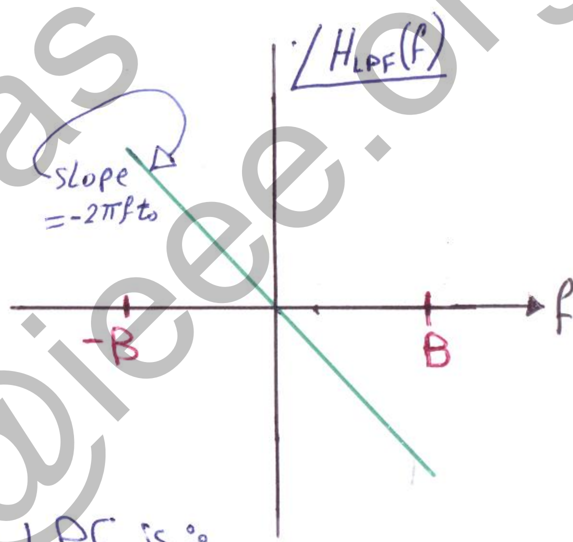


Low Pass Filter (LPF) [ideal]

$$H_{LPF} = H_0 \Pi\left(\frac{f}{2B}\right) e^{-j2\pi f t_0}$$

where H_0 is constant, H_0 stands for the passband gain.

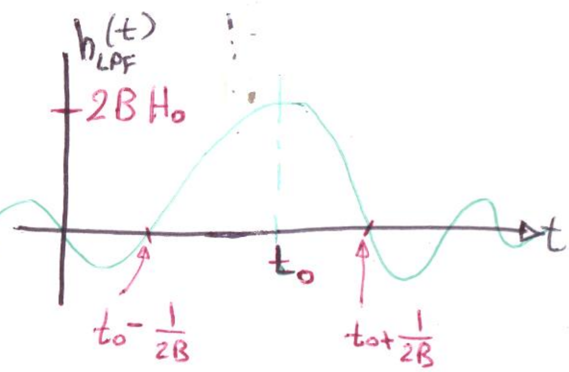




* The impulse response of an ideal LPF is:

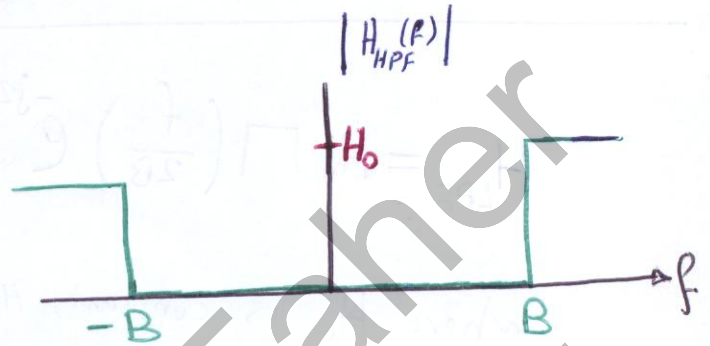
$$h_{LPF}(t) = 2B H_0 \, \mathrm{sinc}[2B(t-t_0)]$$

therefore, they call the sinc function as the filter function or the interpolation function.



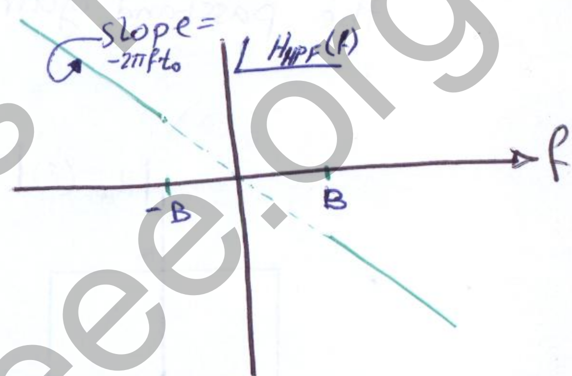
High Pass Filter (HPF) [ideal]

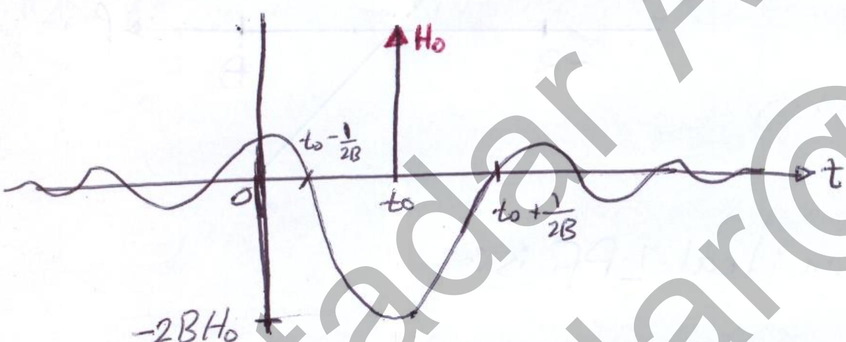
$$H_{HPF}(f) = H_o\left[1 - \Pi\left(\frac{f}{2B}\right)\right] e^{-j2\pi f t_o}$$



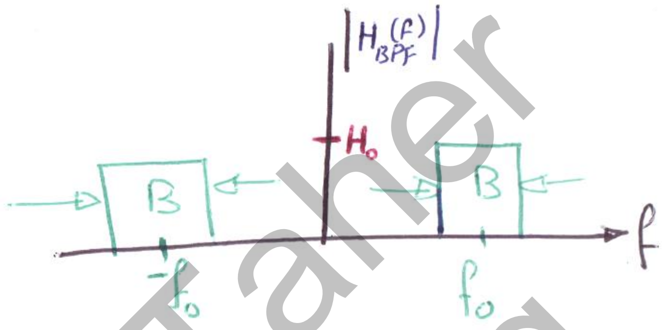
$$h_{HPF}(t) = H_o\{\delta(t-t_o) - 2B\,\text{sinc}[2B(t-t_o)]\}$$

Sketch - Homework



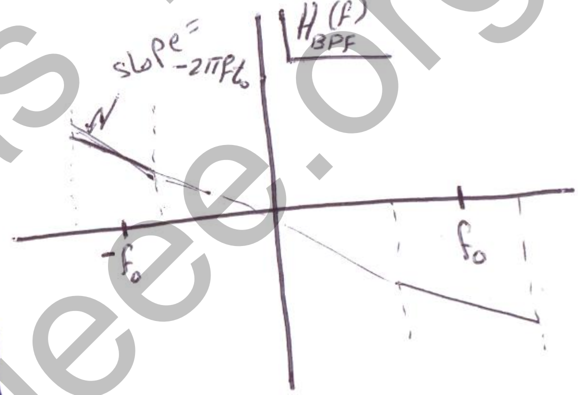


Band Pass Filter (BPF) [ideal]



$$H_{BPF}(f) = \left[H_L(f-f_o) + H_L(f+f_o)\right] e^{-j2\pi f t_o}$$

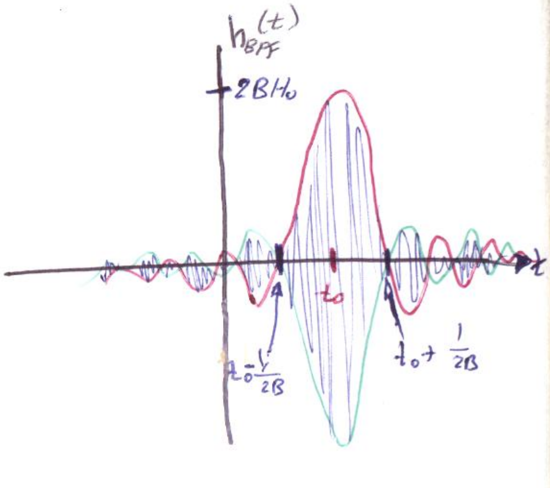
where $H_L = H_o \Pi\left(\frac{f}{B}\right)$



$$\therefore h_{BPF}(t) = 2 h_L(t-t_o)\cos(2\pi f_o(t-t_o))$$

or

$$h_{BPF}(t) = 2BH_o \operatorname{sinc}\left[B(t-t_o)\right] \cos\left\{2\pi f_o(t-t_o)\right\}$$



Sampling Theory

The continuous time signal $x(t)$ can be sampled instantinuously as

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(t - nT_s) \quad (1)$$

where T_s is the sampling interval.

* After Sampling:

① Is it possible to recover $x(t)$ from $x_s(t)$?

② How is $x(t)$ recovered from $x_s(t)$?

* To answer these two questions, we have to understand the uniform Sampling theorem.

Before we go ahaed, Lets have some important Knowledge about signals and their types.

* In real world, there are continuous-time signals and discrete-time signals

* Due to the recent advance developement in digital technology:-

  - Inexpensive,
  - Light weight,
  - programmable, and
  - easily reproducible

  discrete-time systems are available.

→ therefore, the processing of discrete-time signals is more flexible and is also preferable to processing than the continuous-time ones.

* Thus, we should be able to convert a continuous-time signals into discrete-time-signals.

* Sampling process, is a tool that can be used to convert the continuous-time signal to a discrete-time signal and vice-versa.

Sampling Theory :

A band limited signal of bandwidth W Hz sampled at frequency f_s ; can be reconstructed from the sampled version if $f_s \geqslant 2W$.

Proving Sampling Theory :

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(t-nT_s) \quad (1)$$

$T_s = \frac{1}{f_s}$ = sampling interval.

Since we know $\delta(t-nT_s)$ is zero everywhere except at $t = nT_s$

$$\therefore x_\delta(t) = \sum_{n=-\infty}^{\infty} x(t)\,\delta(t-nT_s) = x(t)\sum_{n=-\infty}^{\infty}\delta(t-nT_s) \quad (2)$$

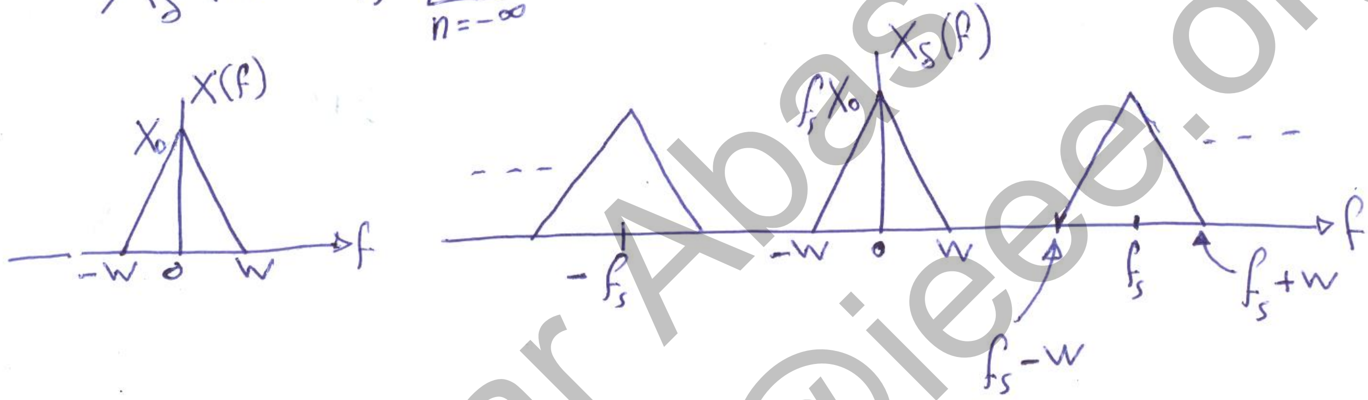
$$X_\delta(f) = X(f) * \left[f_s \sum_{n=-\infty}^{\infty}\delta(f-nf_s)\right] \quad (3)$$

Since $X(f) * \delta(f - nf_s) = \int_{-\infty}^{\infty} X(u)\, \delta(f - u - nf_s)\, du$

$\therefore \; X(f) * \delta(f - nf_s) = X(f - nf_s)$ ——— (4)

by the sifting property of the delta function,

$$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s) \quad \text{——— (5)}$$



$\therefore$ Sampling produces a periodic repetition of $X(f)$ in the frequency domain with a spacing f_s.

① If $f_s < 2W$, the separate terms overlap, $x(t)$ can not be recovered from $x_\delta(t)$. (aliasing)

② If $f_s = 2W$, the separate terms touch each other, $x(t)$ can be recovered from $x_\delta(t)$ using a sharp LPF.

③ If $f_s > 2W$, $x(t)$ can be recovered from $x_\delta(t)$ using LPF.

End of Prove ———

* The reconstruction filter should has a bandwidth B in the range

$$\boxed{W < B < (f_s - W)}$$

where W : Bandwidth of the signal $x(t)$

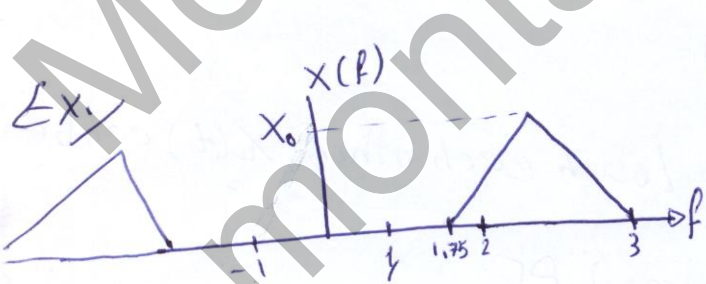
f_s : Sampling frequency

NOTE : Nyquist frequency is the 2W frequency.

Sampling a Bandpass Signal

if a signal has a spectrum W Hz and upper frequency f_u Hz, then a rate f_s at which the signal can be sampled is $2f_u/m$ Hz, where m is the largest integer not exceeding f_u/W.

Ex.



$W = 3 - 1.75 = 1.25$ Hz

$f_u = 3$ Hz

$m \leq \left\lceil \frac{f_u}{W} \right\rceil = \frac{3}{1.25} = 2.4$

$\therefore m = 2$

$\therefore f_s = \frac{2f_u}{m} = \frac{2 * 3}{2} = 3$ Hz.

note that if the signal is a Lowpass signal then, $f_s = 6$ Hz.

EX. A continuous time signal $x(t) = 8\cos(200\pi t)$. Determine:-

① Minimum sampling rate, (Nyquist rate to avoid aliasing).

② If sampling frequency $f_s = 400$ Hz, what is the discrete-time signal $x(n)$ or $x(nT_s)$ obtained after sampling?

③ If sampling frequency $f_s = 150$ Hz, what is the discrete-time signal $x(n)$ or $x(nT_s)$ obtained after sampling?

④ what is the frequency $0 < f < \frac{f_s}{2}$ that yields samples identical to those obtained in part ③?

Solution: $x(t) = 8\cos(2\pi 100 t) \rightarrow f = 100$ Hz.

① $f_s = 2f = 2*100 = 200$ Hz.

② $f_s = 400\text{ Hz} \rightarrow x(nT_s) = x\left(\frac{n}{f_s}\right) = 8\cos\left(2\pi n \frac{100}{400}\right) = 8\cos\left(\frac{2\pi n}{4}\right) = 8\cos\left(\frac{\pi n}{2}\right)$.

$\therefore x(n) = x(nT_s) = 8\cos\left(\frac{n\pi}{2}\right)$.

③ $f_s = 150\text{ Hz} \rightarrow x(nT_s) = x\left(\frac{n}{f_s}\right) = 8\cos\left(2\pi n \frac{100}{150}\right) = 8\cos\left(\frac{4\pi n}{3}\right) = 8\cos\left(\frac{6\pi n}{3} - \frac{2\pi n}{3}\right)$

$x(n) = 8\cos\left[\left(2\pi - \frac{2\pi}{3}\right)n\right] = 8\cos\left(\frac{2\pi n}{3}\right)$

④ $f_s = 150\text{ Hz} \rightarrow x(nT_s) = x\left(\frac{n}{f_s}\right) = 8\cos(2\pi f t) = 8\cos(2\pi f n/150)$

From part ③ $\frac{2\pi n}{3} = \frac{2\pi f n}{150} \rightarrow f = \frac{150}{3} = 50$ Hz

$\therefore x_4(t) = 8\cos(100\pi t)$.

NOTE: $x(t) = 8\cos(2\pi 100 t)$, if $f_s = 150$, then alis frequency $f_a = 150 - 100 = 50$ Hz will appears.

EX. An analog signal is expressed by the equation $x(t) = 3\cos(50\pi t) + 10\sin(300\pi t) +$ ~~+10 sin(300πt)~~ $- \cos(100\pi t)$. Calculate the Nyquist rate for this signal.

Solu.

$$x(t) = 3\cos(50\pi t) + 10\sin(300\pi t) - \cos(100\pi t)$$

$$x(t) = 3\cos(\omega_1 t) + 10\sin(\omega_2 t) - \cos(\omega_3 t)$$

$$\omega_1 = 2\pi f_1 = 50\pi \rightarrow f_1 = 25\ Hz$$

$$\omega_2 = 2\pi f_2 = 300\pi f_2 \rightarrow f_2 = 150\ Hz$$

$$\omega_3 = 2\pi f_3 = 100\pi f_3 \rightarrow f_3 = 50\ Hz$$

Largest frequency is $f_2 = 150\ Hz$

$$\therefore f_s = 2 f_2 = 300\ Hz.$$

EX. Find the Nyquist rate and the Nyquist interval for the signal

$$x(t) = \frac{1}{2\pi}\cos(4000\pi t)\cos(1000\pi t).$$

Solu. since $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$$\therefore x(t) = \frac{1}{2\pi}\cdot\frac{2}{2}\cos(4000\pi t)\cos(1000\pi t)$$

$$= \frac{1}{4\pi}\left[2\cos(4000\pi t)\cos(1000\pi t)\right]$$

$$= \frac{1}{4\pi}\left[\cos(4000\pi t + 1000\pi t) + \cos(4000\pi t - 1000\pi t)\right]$$

$$= \frac{1}{4\pi}\left[\cos(5000\pi t) + \cos(3000\pi t)\right]$$

$$\therefore \omega_1 = 2\pi f_1 = 5000\pi f_1 \longrightarrow f_1 = 2500\ Hz$$

$$\omega_2 = 2\pi f_2 = 3000\pi f_2 \longrightarrow f_2 = 1500\ Hz$$

$$\therefore f_s = 2 f_1 = 5000\ Hz$$

$$T_s = \frac{1}{f_s} = \frac{1}{5000} = 0.2 * 10^{-3}\ sec.$$

Reconstruction Filter (LPF) :

* reconstruction filter is a low-pass-filter (LPF), which is also known as interpolation filter.

* The process of reconstructing the signal from the discret-time version is called interpolation.

- Assuming an ideal LPF :

$$\boxed{H(f) = H_0 \Pi\left(\frac{f}{2B}\right) e^{-j2\pi f t_0}} \quad W \leq B \leq f_s - W$$

$$\therefore Y(f) = f_s H_0 X(f) e^{-j2\pi f t_0}$$

and the time-domain becomes :-

$$\boxed{y(t) = f_s H_0 x(t - t_0)}$$ — reconstructed signal.

OR $$y(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\, h(t - nT_s)$$

$$y(t) = 2BH_0 \sum_{n=-\infty}^{\infty} x(nT_s) \operatorname{sinc}[2B(t - t_0 - nT_s)]$$

But $B = \frac{f_s}{2}$, $H_0 = T_s$, and let $t_0 = 0$ for simplicity

$$y(t) = \sum_{n} x(nT_s) \operatorname{sinc}(f_s t - n)$$

143

154