Fundamentals of Communications Engineering

Department of Communications Engineering, College of Engineering, University of Diyala, 2016-2017

Class: Second Year
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Room: Comm-02

Lecture: 12

+ Impulse Response

* mostly the system impulse response is denoted by $h(t)$.

$$h(t) \triangleq F\{\delta(t)\} \quad -(1)$$ ← impulse response

So,

$$y(t) = \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda$$ ← taking the effect of a system on the i/p signal.

+ Step Response

$$g(t) \triangleq F\{u(t)\} \quad , \quad g(t) = h(t) * u(t)$$

from which

$$h(t) = \frac{dg(t)}{dt}$$

Transfer Function

* The F.T. of $y(t) = x(t) * h(t)$ is

$$Y(f) = X(f)\,H(f)$$

here $H(f)$ is called the transfer Function or it is called Frequency Response of the system having impulse response $h(t)$.

$$\therefore \boxed{H(f) = \frac{Y(f)}{X(f)}}$$

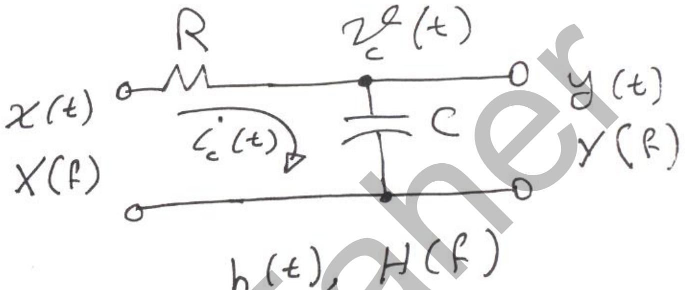
The i/P/o/P relationships for spectral Densities $\circ\circ$

$$G_y(f) = |Y(f)|^2 = |X(f)\,H(f)|^2 = |H(f)|^2\,G_x(f)$$

$$S_y(f) = |H(f)|^2\,S_x(f)$$

Ex. Consider the following circuit, find the system response when $x(t) = A\Pi\left(\frac{t-\frac{T}{2}}{T}\right)$.

- we can find the transfer function $H(f)$ using the AC steady state analysis



RC-Lowpass filter

$$\frac{Y(f)}{X(f)} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$\boxed{H(f) = \frac{1}{1 + j\omega RC}}$$

-we can find $H(f)$ using the circuit differential equation

$$x(t) = i_c(t)R + y(t) \quad \text{—— (1)}$$

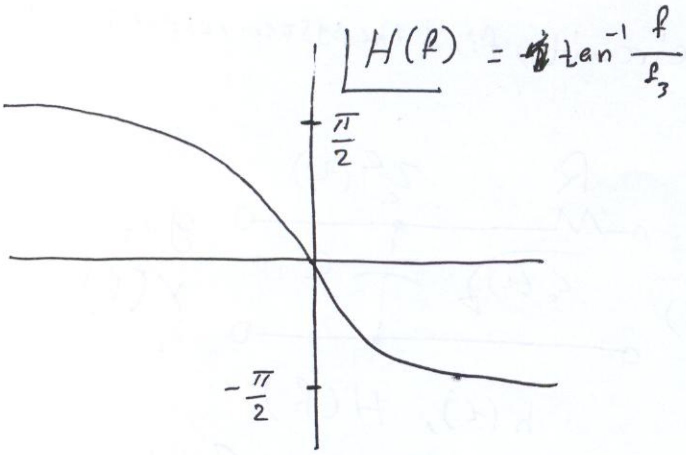
$$i_c(t) = C\frac{dv_c(t)}{dt} = C\frac{dy(t)}{dt} \quad \text{—— (2)}$$

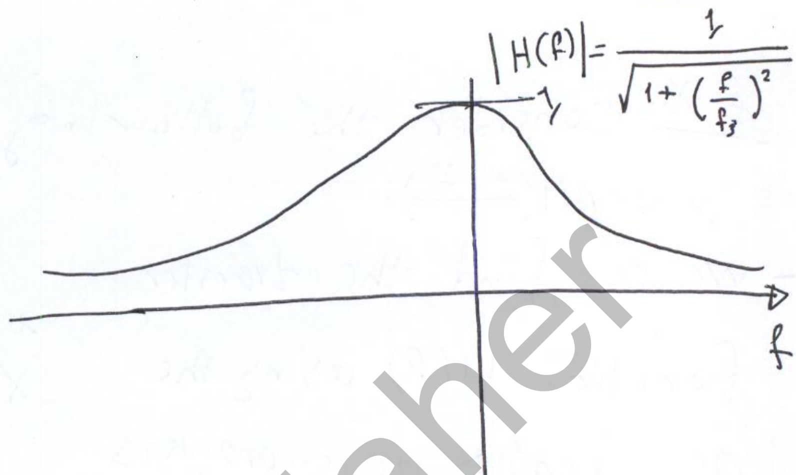
$$\therefore RC\frac{dy(t)}{dt} + y(t) = x(t) \quad \text{—— (3)}$$

taking F.T. for both sides of equ. (3)

$$j2\pi f RC\, Y(f) + Y(f) = X(f)$$

$$\therefore H(f) = \frac{Y(f)}{X(f)} = \frac{1}{1 + j\omega RC}$$





where $f_3 = \dfrac{1}{2\pi RC}$

NOW the Fourier Transform of $x(t)$ is

$$X(f) = F.T.\left\{A\,\Pi\left(\frac{t-\frac{T}{2}}{T}\right)\right\} = AT\,\text{sinc}(fT)\,e^{-j\pi f T}$$

$$\therefore\ Y(f) = X(f)\,H(f) = AT\,\text{sinc}(fT)\left[\frac{1}{1+j\,f/f_3}\right]e^{-j\pi f T}$$

the system response is $y(t) = IFT\{Y(f)\}$

$\rightarrow h(t) = \dfrac{1}{RC}\,e^{-\frac{t}{RC}}\,u(t)$

$\rightarrow$ since $A\,u(t) * e^{-\alpha t}u(t) = \dfrac{A}{\alpha}\left[1-e^{-\alpha t}\right]u(t)$

$\rightarrow$ since $A\,\Pi\left(\dfrac{t-\frac{T}{2}}{T}\right) = A\left[u(t) - u(t-T)\right]$

and here $\alpha = \dfrac{1}{RC}$, so

$$y(t) = \frac{A}{RC}RC\left[1-e^{-t/(RC)}\right]u(t) - \frac{A}{RC}RC\left[1-e^{-\frac{t-T}{RC}}\right]u(t-T)$$

problems Homework

1) the input $x(t)$ & the impulse response $h(t)$ of a continuous LTI system are given by: $x(t) = u(t)$, $h(t) = e^{-\alpha t} u(t)$, $\alpha > 0$. Compute the output $y(t)$.

Ans. $\frac{1}{\alpha}(1 - e^{-\alpha t})\, u(t)$.

) compute the output $y(t)$ for a continuous-time LTI system whose impulse response $h(t)$ and the Input $x(t)$ are given by: $h(t) = e^{-\alpha t} u(t)$ & $x(t) = e^{\alpha t} u(-t)$, $\alpha > 0$

ns. $\frac{1}{2\alpha} e^{-\alpha |t|}$

) For an LTI system with unit impulse response $h(t) = e^{-2t} u(t)$, determine the response $y(t)$ for the input $f(t) = e^{-t} u(t)$.

ns. $y(t) = (e^{-t} - e^{-2t})\, u(t)$.

) For an LTI system with impulse response $h(t) = 6 e^{-t} u(t)$, determine the system response to the input: ⓐ $2u(t)$ and ⓑ $3 e^{-3t} u(t)$.

ns. ⓐ $12(1 - e^{-t})\, u(t)$ ⓑ $9(e^{-t} - e^{-3t})\, u(t)$.

Repeat problem ④ if the input $f(t) = e^{-t} u(t)$.

Ans. $6t\, e^{-t} u(t)$.

) suppose that the input $x(t) = e^{-t} u(t+2)$ to a system of impulse response $h(t) = e^{t} u(-t)$. Find the esponse $y(t)$

ns. $\frac{e^{4+t}}{2}$ for $-2 < t < \infty$

and $\frac{e^{-t}}{2}$ for $t > 0$

(7) Given an LTI system with the output $y(f) = \frac{1}{1+j2\pi f} - \frac{1}{1+j4\pi f}$, determine the input $x(t)$ if the system's impulse respons is $h(t) = e^{-2t} u(t)$

ans. $x(t) = e^{-t} u(t)$

(8) The input signal $X(f) = \frac{1}{1+j2\pi f}$ is applied to an LTI system whose $h(t) = 1-t$ for $0 \le t \le 1$ and zero otherwise. $h(t) = 1-t$

What is the output of the system $y(t)$?

ans. $y(t) = \begin{cases} 0 & t<0 \\ 2-t-2e^{-t} & 0 \le t \le 1 \\ e^{-(t-1)} - 2e^{-t} & t>1 \end{cases}$

(9) Calculate the system time response of an LTI system of $h(t) = 2$ for $-1 \le t \le 2$ and zero otherwise when the input is $x(t) = 1.5$ for $-2 \le t \le 3$ and zero otherwis

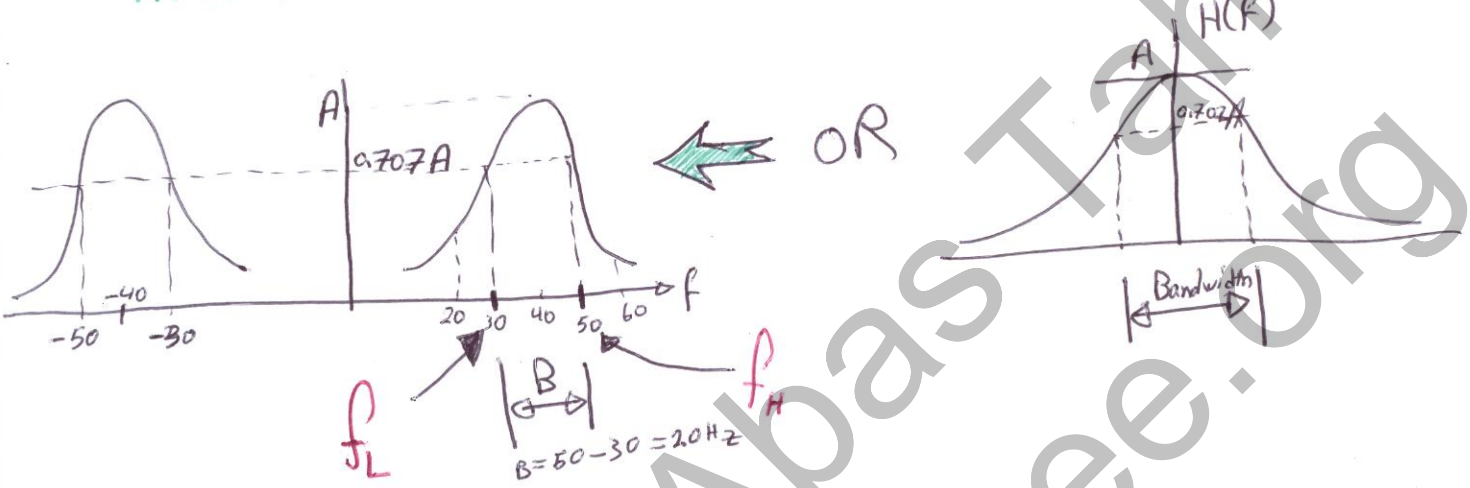
(10) Find $h(t)$ for the system input/output shown below

$h(t) = ?$ $\quad x(t) = e^{-3t} \quad t \ge 0$

$H(f) = ?$ $\quad y(t) = -\frac{1}{3}(e^{-3t} + 1)$

System Bandwidth :

* System Bandwidth B is the difference between the maximum and minimum positive frequencies.



$$\therefore B = f_H - f_L$$

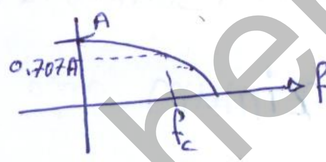
* Cutoff Frequency (f_c) : is the frequency at which $|H(f_c)| = \alpha |H(f)|_{max}$

  * Typical value of $\alpha = \frac{1}{\sqrt{2}} = 0.707$

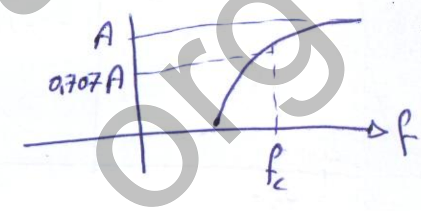
Thus : The frequency at which the $|H(f)| = \frac{|H(f)|_{max}}{\sqrt{2}}$, is called the half-power cutoff frequency or 3 dB cutoff frequency.

Hence : The system bandwidth, B, is the difference between the upper & Lower cutoff frequencies.

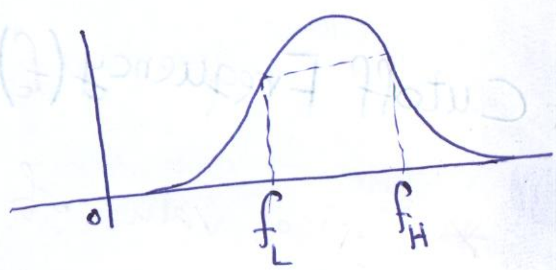
Low Pass System : is the system that has a frequency bandwidth starts at Zero and extends to f_c.



High pass system : is the system that has the first cutoff frequency ~~star~~ beyond Zero Hz

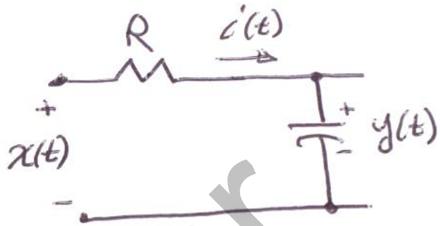


Band Pass System : is the system that consists of two cutoff frequencies, f_L (lower cutoff-frequency) and an upper cutoff-frequency f_H



$$\boxed{B = f_H - f_L}$$

Ex. Find the frequency response for the system given below.



solution

$$R\, i(t) + y(t) = x(t) \quad \text{——(1)}$$

$$i(t) = C\,\frac{dy(t)}{dt} \quad \text{——(2)}$$

using (2) in (1) :- $RC\,\dfrac{dy(t)}{dt} + y(t) = x(t)$

using the differentiation property of Fourier transform :-

$$RC(j2\pi f)\,Y(f) + Y(f) = X(f)$$

$$j2\pi f RC\, Y(f) + Y(f) = X(f)$$

$$\therefore\ Y(f)\,(j2\pi f RC + 1) = X(f)$$

$$\therefore\ H(f) = \frac{Y(f)}{X(f)} = \frac{1}{1 + j2\pi f RC}$$

Ex. LTI BIBO stable system is characterized by the system differential equation

$$\frac{d^3 y(t)}{dt^3} + A\,\frac{d^2 y(t)}{dt^2} + B\,\frac{dy(t)}{dt} + C\,y(t) = D\,\frac{dx(t)}{dt} + E\,x(t)$$

find the system frequency response.

solution $(j2\pi f)^3 Y(f) + A\,(j2\pi f)^2\, Y(f) + B\,(j2\pi f)\,Y(f) + C\,Y(f) = D\,(j2\pi f)\,X(f) + E\,X(f)$

$$\therefore\ H(f) = \frac{Y(f)}{X(f)} = \frac{E + j2\pi D f}{(C - 4\pi^2 A f^2) + j2\pi f\,(B - 4\pi^2 f^2)}$$

Distortionless Transmission :-

* Generally :- The magnitude & the phase of the spectral components will be affected during the processing of the LTI system, according to

$$|Y(f)| = |X(f)|\,|H(f)| \quad \text{——— (1)}$$

$$\angle Y(f) = \angle X(f) + \angle H(f) \quad \text{——— (2)}$$

* A distortionless LTI system must present same attenuation to all spectral components, both for amplitude & phase

$$H_{ideal}(f) = H_0\, e^{-j2\pi f t_0} \qquad \underbrace{f_1 \le f \le f_2}_{\text{i.e., in a specified frequency range.}} \quad \text{——— (3)}$$

Thus,

$$Y(f) = X(f)\,H(f)$$
$$= X(f)\,H_0\, e^{-j2\pi f t_0} \quad \text{——— (4)}$$

and in time-domain (time-response)

$$y(t) = H_0\, x(t - t_0) \quad \text{——— (5)}$$

where H_0 is constant, t_0 is time delay.

∴ The distortionless LTI system produces a delayed version of the input.

Group delay : different types of distortion are present, one of them is the delay distortion.

Delay or Envelope distortion is well known as the group delay. Mathematically it is;

$$\tau_g(f) = \frac{-1}{2\pi}\frac{d\theta(f)}{df} \qquad \text{(1)}$$

since $\theta(f) = \angle H_{ideal}(f) = -2\pi f t_o$

$$\therefore \tau_g(f) = \frac{-1}{2\pi}\frac{d(-2\pi f t_o)}{df}$$

$$\boxed{\tau_g(f) = t_o = \text{constant}}$$

* In practice, a signal will be distorted in passing through some parts of a system.

* Phase or amplitude correction (Equalization) networks must be used to correct this distortion.

* If $\tau_g(f_1) \neq \tau_g(f_2)$ and both f_1 & f_2 within the band of operation

then, there is a delay distortion in the system.

Hence: Group delay is a measure of time delay of the amplitude of the input as a function of frequency

$$|X(f)| \xrightarrow{\text{time delay}} |X(f)| e^{-j2\pi f t_0}$$

$$\boxed{\text{time-Delay} \longleftrightarrow \text{phase shift}}$$

GROUP-DELAY

Phase Delay:

for LTI system, the phase delay can be defined as

$$\tau_p(f) \triangleq \frac{-1}{2\pi f} \angle H(f) \qquad (1)$$

* In a linear phase LTI system

$$\tau_p(f) = \frac{-1}{2\pi f}(-2\pi f t_0)$$

$$\boxed{\tau_p(f) = t_0 = \text{constant}}$$

Thus, in general: $\boxed{\tau_p(f) = \frac{-\theta(f)}{2\pi f}}$

Hence: Phase delay is a measure of the time delay of the phase of the input signal.

$$\boxed{\angle X(f) \xleftrightarrow{\text{time delay}} \tau_p(f)}$$

Distortion Types :-

After understanding group & phase delays, we can conclude the following types of distortion using this equation

$$H(f) = H_0 \, e^{-j2\pi f t_0}$$

(1) Amplitude distortion :

amplitude response is not constant over a frequency band of interest.

(2) Phase distortion :

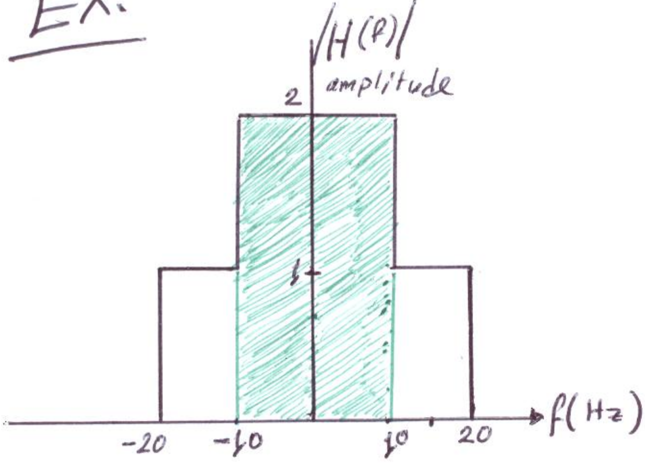
phase response is not linear over a frequency band of interest.

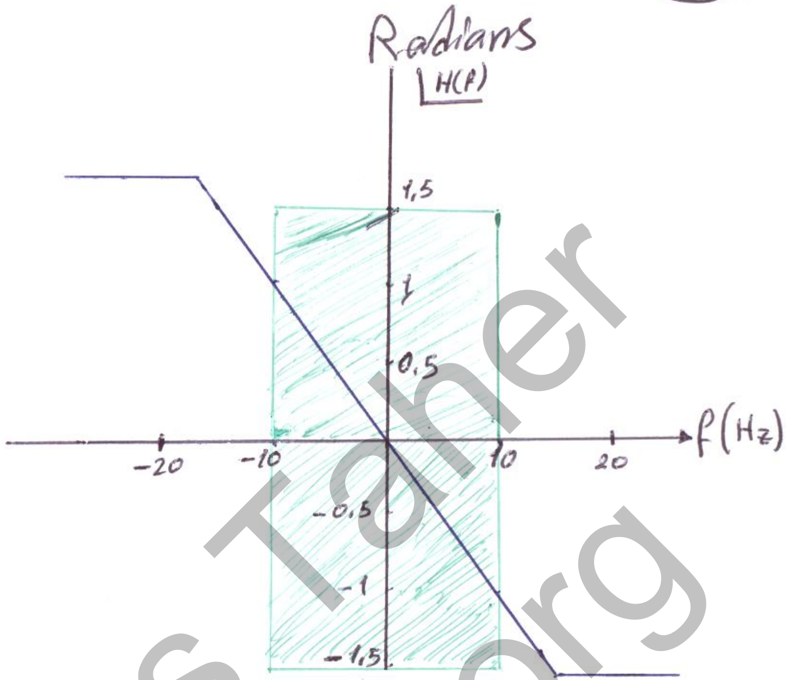
(3) Nonlinear distortion :

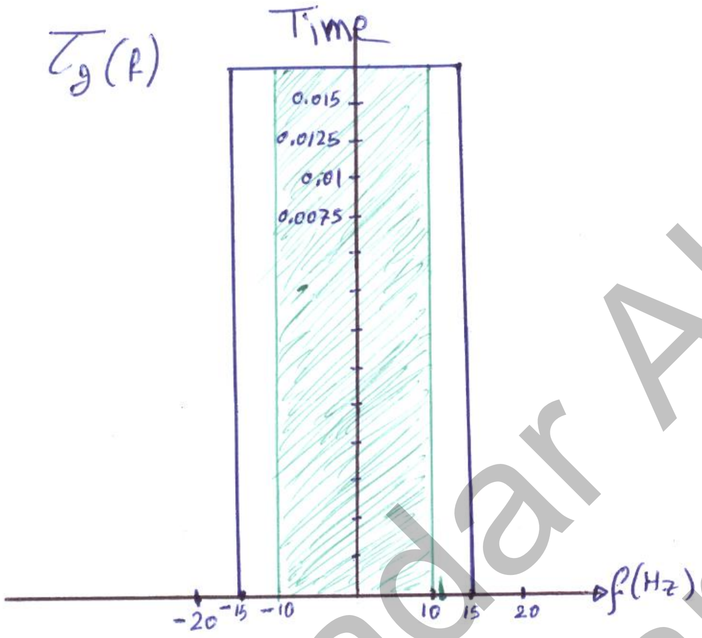
The system is non-linear.

ex. $y(t) = k_0 + k_1 x(t) + k_2 x^2(t)$.

Ex.







* For signals with $|f| < 10\,Hz$, there is no distortion, amplitude or phase/group delay.

* For $|f| > 15\,Hz$ both amplitude and phase distortion are present.

* For $10 < |f| < 15$ amplitude distortion are present.

*

NonLinear distortion :

A non-Linear system may be expressed as

$$y(t) = a_0 + a_1 x(t) + a_2 x^2(t) + \cdots = \sum_{n=0}^{\infty} a_n x^n(t) \quad \text{—— (1)}$$

Let's assume

$$y(t) = a_1 x(t) + a_2 x^2(t) \quad \text{—— (2)}$$

if $\quad x(t) = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t) \quad \text{—— (3)}$

use (2) in (1) :-

$$y(t) = a_1 \left[A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)\right] + a_2 \left[A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)\right]^2$$

① $\longrightarrow \quad = a_1 \left[A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)\right]$

② $\longrightarrow \quad + \left\{ \frac{a_2}{2}(A_1^2 + A_2^2) + \frac{a_2}{2}\left[A_1^2 \cos(2\omega_1 t) + A_2^2 \cos(2\omega_2 t)\right] \right\}$

③ $\longrightarrow \quad + a_2 A_1 A_2 \left\{ \cos\left[(\omega_1 + \omega_2)t\right] + \cos\left[(\omega_1 - \omega_2)t\right] \right\}$

* the first line is the required output,
* the second line is termed harmonic distortion, and
* the third line is termed intermodulation distortion.

Hence, in general ↘

In general:- if $y(t) = a_1 x(t) + a_2 x^2(t)$

$$y(t) = a_1 x(t) + a_2 \underbrace{x(t) \cdot x(t)}_{\text{multiplication} \overset{F.T.}{\longleftrightarrow} \text{convolution.}}$$

$$Y(f) = a_1 X(f) + a_2 X(f) * X(f)$$

EX. if you have a system of the form $y(t) = a_1 x(t) + a_2 x^2(t)$, find the output frequency-domain if the input signal is

$$X(f) = A \Pi\left(\frac{f}{2W}\right)$$

Solution

$$Y(f) = a_1 X(f) + a_2 X(f) * X(f)$$

$$Y(f) = a_1 A \Pi\left(\frac{f}{2W}\right) + a_2 A^2 \left[\Pi\left(\frac{f}{2W}\right) * \Pi\left(\frac{f}{2W}\right)\right]$$

$$= a_1 A \Pi\left(\frac{f}{2W}\right) + a_2 2W A^2 \Lambda\left(\frac{f}{2W}\right)$$