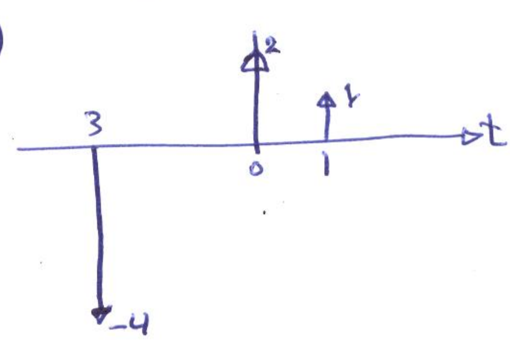
EX. 1 Find the Fourier transform of $g(t) = \delta(t-1) + 2\delta(t) - 4\delta(t+3)$.

Solution $g(t) = \delta(t-1) + 2\delta(t) - 4\delta(t+3)$



we know $\delta(t - t_0) \xleftrightarrow{FT} e^{-j2\pi f t_0}$

and using the linearity property

$$\therefore\ G(f) = e^{-j2\pi f} + 2 - 4\, e^{j2\pi f 3}$$

EX. 2 Find the Fourier transform of $g(t) = A\,\mathrm{rect}\left(\frac{t-7}{T}\right) + \delta(t+7)$

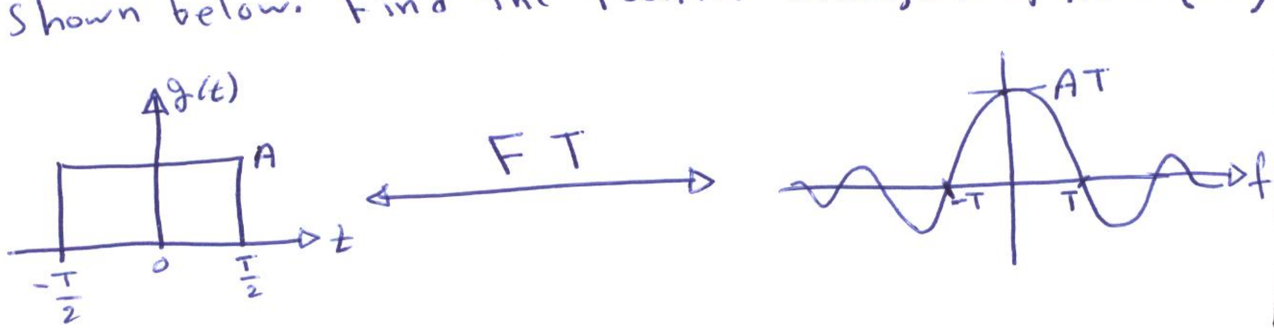
Solution $g(t) = A\,\mathrm{rect}\left(\frac{t-7}{T}\right) + \delta(t+7)$

using linearity property

$$G(f) = FT\left\{A\,\mathrm{rect}\left(\frac{t-7}{T}\right)\right\} + FT\left\{\delta(t+7)\right\}$$

$$G(f) = AT\,\mathrm{sinc}(fT) + e^{j14\pi f}$$

EX.3 given the signal and its frequency-domain sketches shown below. Find the Fourier transform of $A\,\text{sinc}(2Wt)$



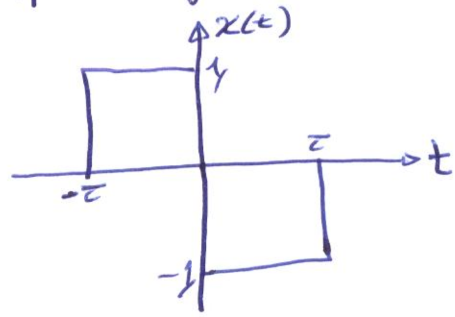
Solution $g(t) = A\,\text{rect}\left(\frac{t}{T}\right) \xleftrightarrow{FT} G(f) = AT\,\text{sinc}(fT)$

Using duality theorem:

$$G(t) \longleftrightarrow g(-f)$$

$$A\,\text{sinc}(2Wt) \longleftrightarrow \frac{A}{2W}\,\text{rect}\left(\frac{-f}{2W}\right) = \frac{A}{2W}\,\text{rect}\left(\frac{f}{2W}\right).$$

EX.4 Determine the Fourier transform of the signal shown below.



Solution $x(t) = \text{rect}\left(\frac{t+\frac{\tau}{2}}{\tau}\right) - \text{rect}\left(\frac{t-\frac{\tau}{2}}{\tau}\right)$

$$X(f) = \tau\,\text{sinc}(f\tau)\,e^{j2\pi f\frac{\tau}{2}} - \tau\,\text{sinc}(f\tau)\,e^{-j2\pi f\frac{\tau}{2}}$$

$$= \tau\,\text{sinc}(f\tau)\left[e^{j2\pi f\frac{\tau}{2}} - e^{-j2\pi f\frac{\tau}{2}}\right]$$

$$= \frac{2j}{\pi f}\sin^2(\pi f\tau).$$

EX.6 $x(t)$ is a time-domain signal. If $x(t)$ shifted in frequency by $e^{j2\pi f_0 t}$, find the Fourier transform of the above signal combination.

Solution: This is frequency shift or translation property.

$$\int_{-\infty}^{\infty} x(t)\, e^{j2\pi f_0 t}\, e^{-j2\pi f t}\, dt = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi (f-f_0)}\, dt$$

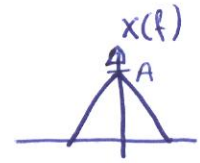
$$\text{so, } FT\left\{x(t)\, e^{j2\pi f_0 t}\right\} = X(f-f_0)$$

EX.7 Find the Fourier transform of $x(t)\cos(2\pi f_0 t)$.

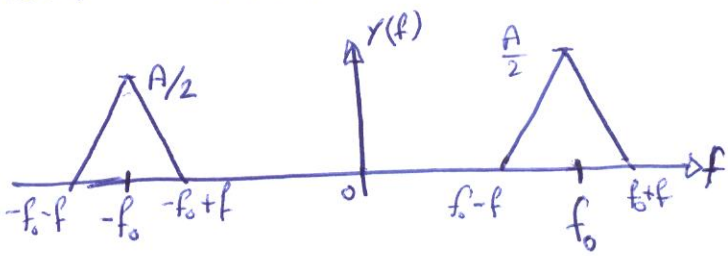
Solution we know $\cos(2\pi f_0 t) = \frac{1}{2} e^{j2\pi f_0 t} + \frac{1}{2} e^{-j2\pi f_0 t}$

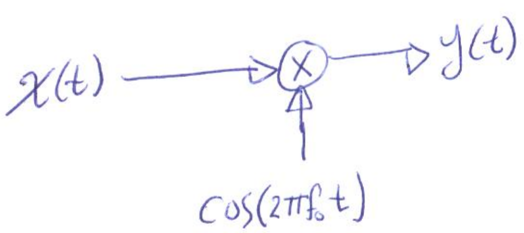
$$\text{then } FT\left\{\frac{1}{2}x(t)\, e^{j2\pi f_0 t} + \frac{1}{2}x(t)\, e^{-j2\pi f_0 t}\right\} = \frac{1}{2}X(f-f_0) + \frac{1}{2}X(f+f_0)$$

For more explanation assume $x(t) \xrightarrow{FT} X(f) \Rightarrow$



then $FT\{x(t)\cos(2\pi f_0 t)\} \Rightarrow$





} A simple modulator