EE6

Example: Convert the following from rectangular to polar form.

$$\bar{C} = -6 + j3$$

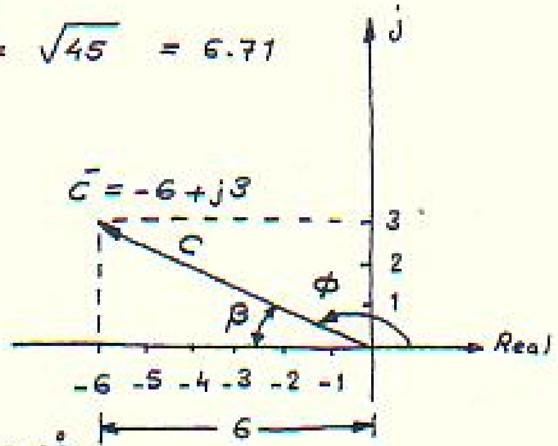
Solution:

$$C = \sqrt{(-6)^2 + (3)^2} = \sqrt{45} = 6.71$$

$$\theta = \tan^{-1}\left(\frac{3}{6}\right)$$

$$= 26.57^\circ$$

$$\Rightarrow \phi = 180^\circ - 26.57^\circ$$
$$= 153.43^\circ$$



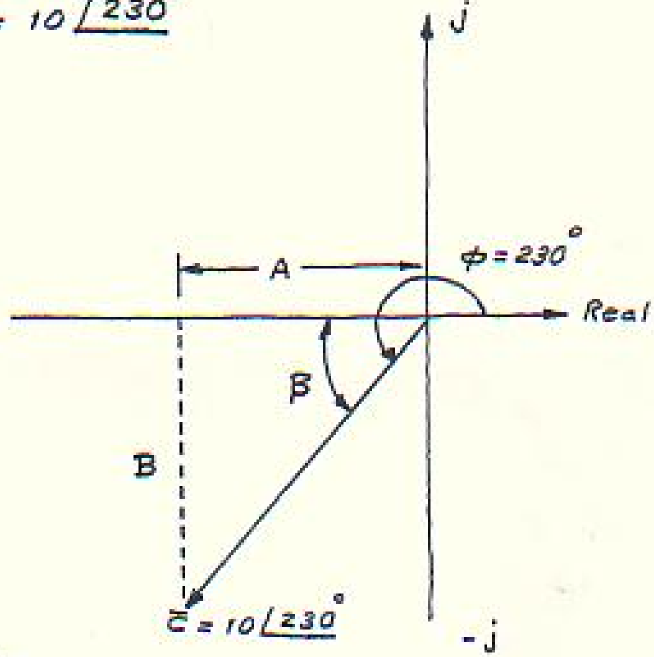
$\therefore$ $\bar{C}$ in polar form is:

$$\bar{C} = C\angle\phi = 6.71\angle 153.43^\circ$$

Example: Convert from polar to rectangular form.

$$\bar{C} = 10\angle 230^\circ$$

Solution:



$$A = 10\cos\beta$$
$$= 10\cos(230^\circ - 180^\circ)$$
$$= 10\cos 50^\circ$$
$$= 6.428$$

$$B = 10\sin\beta$$
$$= 10\sin(230 - 180^\circ)$$
$$= 10\sin 50^\circ$$
$$= 7.66$$

$\therefore$ $\bar{C}$ in rectangular form is

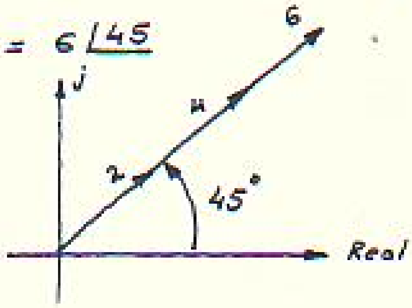
$$\bar{C} = -6.428 - j\,7.66$$

EE6

Mathematical Operations in the Polar Form :

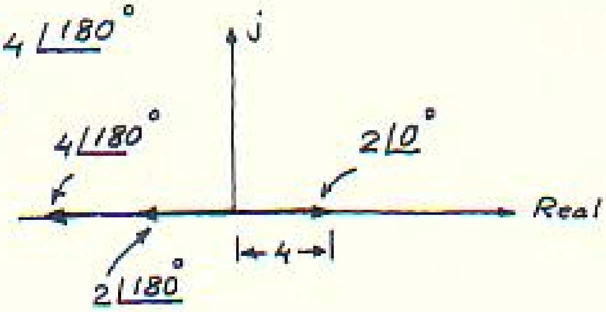
* Addition and Subtraction : Addition or subtraction cannot be performed in polar form unless the complex numbers have the same angle ϕ or differ only by multiples of 180°

Ex: $2\angle 45^\circ + 4\angle 45^\circ = 6\angle 45$



Ex: $2\angle 0^\circ + 4\angle 180^\circ$

$= 2\angle 180^\circ$



* Multiplication : If we have two complex numbers

$$\bar{C}_1 = C_1\angle\phi_1$$
$$\bar{C}_2 = C_2\angle\phi_2$$

Then $\bar{C}_1 \cdot \bar{C}_2 = C_1 C_2 \angle \phi_1+\phi_2$

Ex : Find $\bar{C}_1 C_2$, if $\bar{C}_1 = 5\angle 20^\circ$, and $\bar{C}_2 = -10\angle 30^\circ$

$$\bar{C}_1 \cdot \bar{C}_2 = (5)(-10)\angle 20^\circ + 30^\circ = \underline{-50\angle 50^\circ}$$

* Division : If we have two complex numbers $\bar{C}_1 = C_1\angle\phi_1$ and $\bar{C}_2 = C_2\angle\phi_2$,

Then $\dfrac{\bar{C}_1}{\bar{C}_2} = \dfrac{C_1}{C_2}\angle\phi_1 - \phi_2$

Ex : Given $\bar{C}_1 = 15\angle 10^\circ$ and $\bar{C}_2 = 2\angle 7^\circ$, find $\dfrac{\bar{C}_1}{C_2}$

$$\frac{C_1}{C_2} = \frac{15}{2}\angle 10^\circ - 7^\circ = \underline{7.5\angle 3^\circ}$$

7. Series & Parallel AC Circuits

EE7

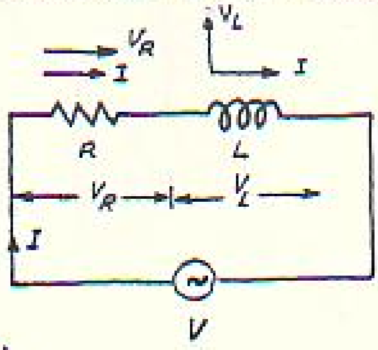
7.1 Series AC Circuits :

7.1.1 AC Through R and L

: Consider the circuit shown below;

V = the rms value of the applied voltage.

I = the rms value of the resultant current.



$\bar{V} = \bar{V}_R + \bar{V}_L$

$\Rightarrow V_R = IR$ (in phase with I)

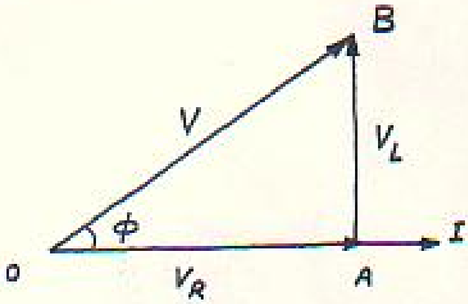
$V_L = IX_L$ (leading I by 90°)

The vector diagram for these voltage drops can be obtained as:

$$\Rightarrow V = \sqrt{V_R^2 + V_L^2}$$

$$= \sqrt{(IR)^2 + (IX_L)^2}$$

$$= I\sqrt{R^2 + X_L^2}$$



voltage triangle

$$\Rightarrow I = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{V}{Z_T}$$

$\Rightarrow$ Z is known as the impedance of the circuit

$$Z_T = \sqrt{R^2 + X_L^2}$$

$$Z_T^2 = R^2 + X_L^2$$

Impedance traingle

* The phase difference angle ϕ can be determined as:

$$\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{\omega L}{R}$$

ويمكن

$\cos\phi = \frac{R}{Z_T}$

$$\therefore \boxed{\phi = \tan^{-1}\frac{X_L}{R}}$$

It is clear that the current I lags behind the applied voltage V by an angle (ϕ), then if :

$v = V_m \sin\omega t \Rightarrow i = I_m \sin(\omega t - \phi)$

$\Rightarrow v_R = I_m R \sin(\omega t - \phi)$

where $I_m = \frac{V_m}{Z}$

$\Rightarrow v_L = I_m X_L \sin(\omega t - \phi + 90^\circ)$

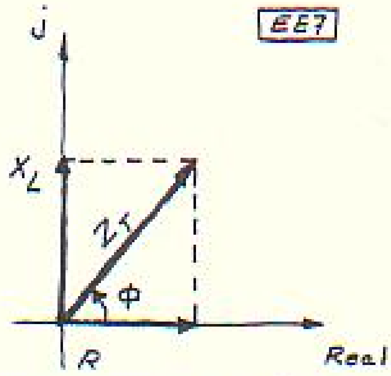
EE7

* In phasor notation

$$\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2$$

$$= R\angle 0 + X_L \angle 90$$

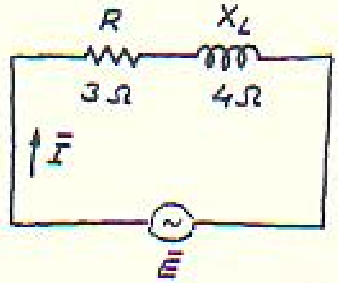
$$\Rightarrow \therefore \bar{Z}_T = R + jX_L$$



Impedance diagram

Example

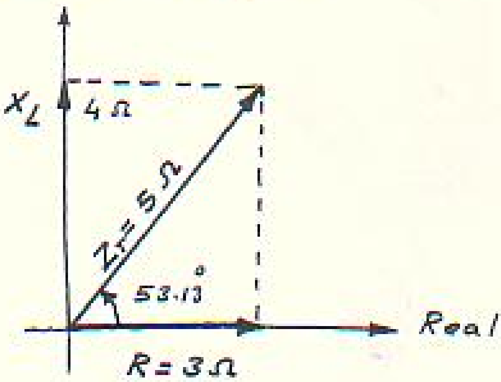
For the circuit shown, determine the total impedance and draw the impedance diagram



Solution : $\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 = R\angle 0 + X_L\angle 90^\circ = R + jX_L$

$$= 3 + j4$$

$$\Rightarrow \therefore \bar{Z}_T = 5\angle 53.13^\circ$$



Note that, the angle Φ (= 53.13° in this example) is always positive in the impedance diagram.

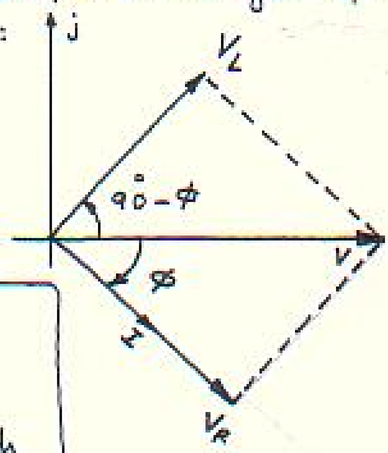
$$\Phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{4}{3}$$

$$= 53.13^\circ$$

The phasor diagram

: The phasor diagram of the voltages of the RL series circuit can be as shown:

$$\bar{V} = \bar{V}_R + \bar{V}_L = \bar{I}R + \bar{I}\bar{X}_L$$
$$= IR\angle{-\phi} + IX_L\angle{90-\phi}$$



phasor diagram

كذلك تسمى بالقدرة الحقيقية (true power)

> The total power (average power) in (Watts) delivered to the circuit is given by the product of V and the component of the current I which is in phase with the applied voltage V

⇒ Then:

$$\boxed{P_T = VI\cos\phi} \qquad \text{Watts}$$

⇐ where $\cos\phi$ is the power factor

* The power factor

: The power factor can be defined as the cosine of the angle (lead or lag) between the current and voltage;

⇒ Power factor = $\cos\phi$

② or it may be defined as the ratio $\frac{R}{Z}$
(see the impedance triangle)

③ or it may be defined as the ratio $\frac{\text{true power}}{\text{apparent power}}$

⇒ Power factor = $\frac{\text{true power}}{\text{apparent power}}$ ⇒ $\frac{\text{Watts}}{\text{Volt-ampere}}$

* Active and Reactive Components of the Circuit Current

:

* The active component is that which is in phase with the applied voltage V, i.e ($I\cos\phi$); it is also called the wattful component.

from the phasor diag. ⇒

* The reactive component is that which is in quadrature with V, i.e ($I\sin\phi$); it is also called the wattless component or the idle component.

⇒ According to these definitions, we have also two components of power each relating its corresponding current component

EE7

* Active, Reactive and Apparent Power (The Power Triangle):

* The Apparent Power:

It is the product of the rms values of the applied voltage and the circuit current

$$\Rightarrow \text{Apparent power} = S = VI = (IZ).I = I^2Z$$

وحدة قياس
volt-ampere (VA)

* The Active Power:

It is the power which is actually dissipated in the circuit resistance.

$$\Rightarrow \text{Active power} = P = I^2R = VI\cos\phi \qquad \text{watts (W)}$$

* The Reactive Power:

It is the power developed in the <u>inductive</u> reactance of the circuit.

$$\Rightarrow \text{Reactive Power} = Q = I^2X_L = I^2(Z\sin\phi)$$

$$\therefore Q = VI\sin\phi$$

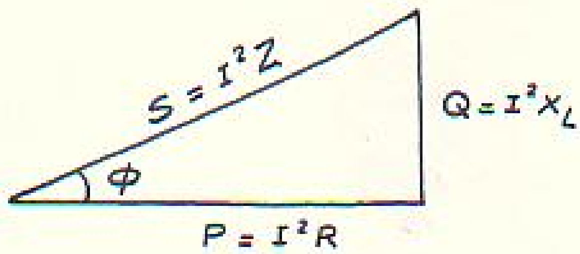
وحدة قياس
Volt-amper reactive (VAR)

These three powers are shown in the power triangle:
From the power triangle:

$$S^2 = P^2 + Q^2$$

or

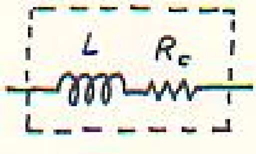
$$S = \sqrt{P^2 + Q^2}$$



The power triangle

* The quality factor of the Coil:

It is defined as the reciprocal of the power factor of the coil. Hence:



coil

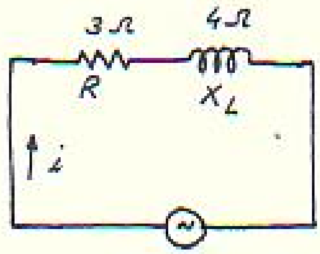
$$\Rightarrow Q_{factor} = \frac{1}{\text{power factor}} = \frac{1}{\cos\phi} = \frac{Z}{R_c} = \frac{\sqrt{R_c^2 + X_L^2}}{R_c}$$

if R_c is too small compared with X_L, then:

$$\boxed{Q_{factor} = \frac{X_L}{R_c}}$$

EE7

Example: For the circuit shown, draw the phasor diagram of the voltages across each element and the applied voltage, and determine :- The power factor.
- The active and reactive power.
- The apparent power.



$v = 141.4 \sin \omega t$

Solution:

* $\because v = 141.4 \sin \omega t \Rightarrow \bar{V} = 100 \angle 0^\circ$

* $\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 = R + jX_L$
$= 3 + j4$
$= 5 \angle 53.13^\circ$

* The current $\bar{I}$ $\Rightarrow \bar{I} = \dfrac{\bar{V}}{\bar{Z}_T}$

$$\therefore \bar{I} = \frac{100 \angle 0^\circ}{5 \angle 53.13^\circ} = 20 \angle -53.13^\circ$$

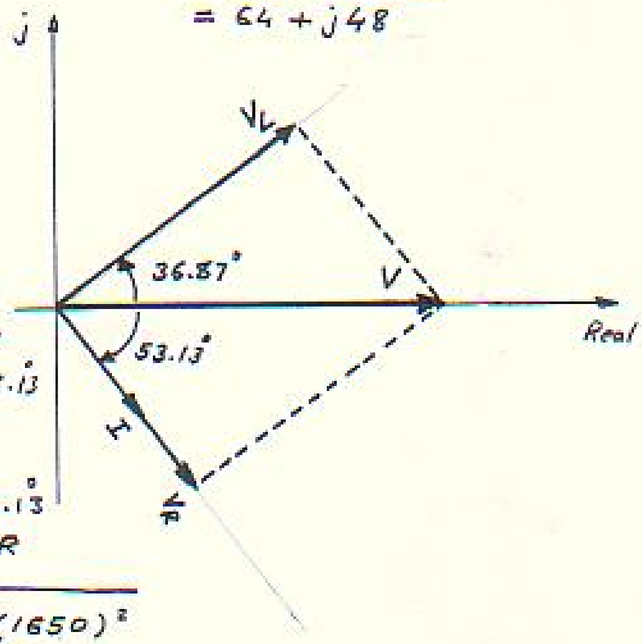
* The voltage drops $\bar{V}_R$ and $\bar{V}_L$:

$\bar{V} = \bar{V}_R + \bar{V}_L$ $\Rightarrow$ $\bar{V}_R = \bar{I}R = (20 \angle -53.13^\circ)(3)$
$= 60 \angle -53.13^\circ$
$= 36 - j48$

$\therefore \bar{V} = 36 - j48 + 64 - j48$ $\Leftarrow$ $V_L = \bar{I}\bar{X}_L = (20 \angle -53.13)(4 \angle 90^\circ)$
$= 100$ $= 80 \angle 36.87^\circ$
$= 100 \angle 0^\circ$ $= 64 + j48$

في الحقيقة يساوي

* The phasor diagram



* **Powers**

active power (real) (average)

$P = I^2R = (20)^2(3) = 1200\ W$

or $P = VI\cos\phi = (100)(20)\cos 53.13^\circ$
$= 1200\ W = 1.2\ kW$

reactive power

$Q = VI\sin\phi = (100)(20)\sin 53.13^\circ$
$= 1600\ VAR = 1.6\ kVAR$

$S = \sqrt{P^2 + Q^2} = \sqrt{(1200)^2 + (1650)^2}$
$= 1968\ VA = 1.968\ kVA$

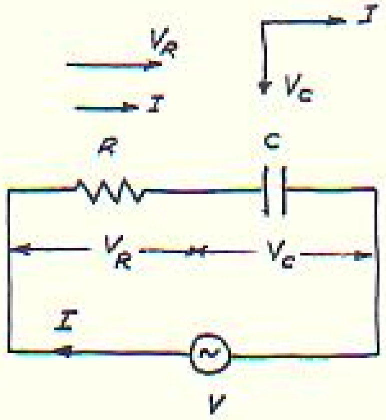
* The power factor: $P.f = \cos\phi = \cos 53.13^\circ = 0.6$ lagging

اعتماداً على التيار بالنسبة للفولتية

7.1.2 AC Through R and C

EE7

Consider the circuit shown, where



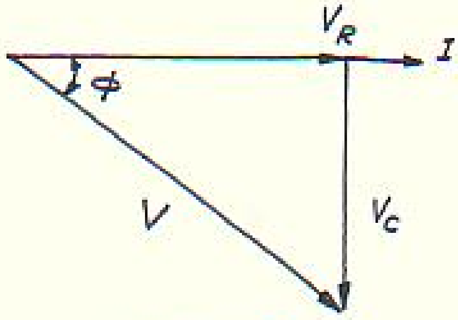
V = rms value of the applied voltage.
I = rms value of the resultant current.

$$V_R = IR \quad \text{(in phase with } I\text{)}$$
$$V_L = IX_c \quad \text{(lagging } I \text{ by } 90^\circ\text{)}$$

* In Vector Notations:

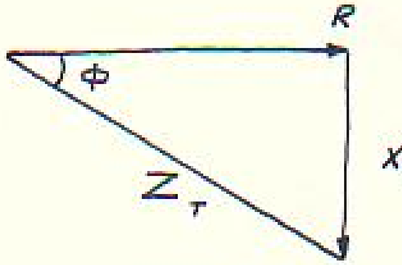
$$\bar{V} = \sqrt{\bar{V}_R^2 + \bar{V}_c^2}$$
$$= \sqrt{I^2R + I^2X_c^2}$$
$$= I\sqrt{R^2 + X_c^2}$$

or
$$I = \frac{V}{\sqrt{R^2 + X_c^2}} = \frac{V}{Z}$$
$$Z = \sqrt{R^2 + X_c^2}$$



voltage triangle

* $$\phi = \tan^{-1}\frac{X_c}{R}$$
$$\therefore \text{P.f} = \cos\phi = \frac{R}{Z}$$



impedance triangle

It is clear that I lead V by an angle ϕ. Hence if:

$$v = V_m \sin \omega t$$

then:

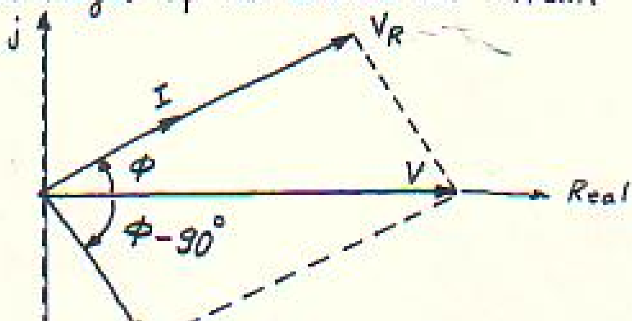
$$i = I_m \sin(\omega t + \phi)$$

so that the current i lead the applied voltage v by an angle ϕ, and

$$v_R = I_m R \sin(\omega t + \phi)$$
$$v_c = I_m X_c \sin(\omega t + \phi - 90^\circ)$$

* The phasor diagram of the voltages of the series RC circuit can be as shown:

$$\bar{V} = V\angle 0$$
$$\bar{I} = I\angle\phi$$
$$\bar{V}_R = V_R\angle\phi$$
$$\bar{V}_c = V_c\angle\phi - 90^\circ$$



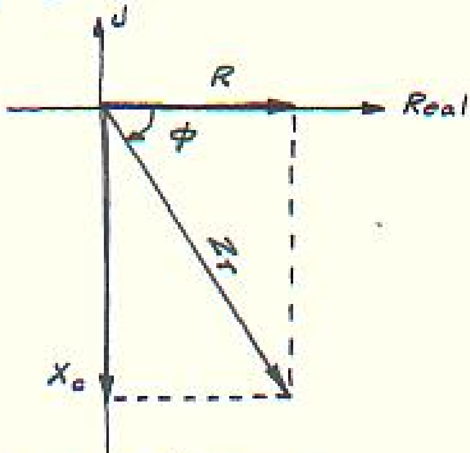
phasor diagram

* **The impedance diagram**

in phasor notation

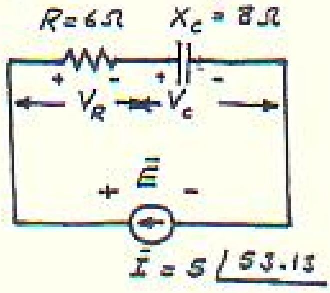
$$\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 = R\angle 0^\circ + X_c\angle -90^\circ$$
$$= R - jX_c$$

$$\therefore \bar{Z}_T = \sqrt{R^2 + X_c^2}$$
$$= Z_T \angle \phi$$



⇒ Note that ϕ is always negative for RC circuits.

Example: For the circuit shown, draw the phasor diagram.



Solution:

* $\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2$
$= 6\angle 0^\circ + 8\angle -90^\circ$
$= 6 - j8 = 10\angle -53.13^\circ$

* $\bar{E}$?

$$\bar{E} = \bar{I}\bar{Z}_T = (5\angle 53.13^\circ)(10\angle -53.13^\circ)$$
$$= \underline{50\angle 0}$$

$\bar{V}_R$?

$$\bar{V}_R = \bar{I}R = (5\angle 53.13^\circ)(6)$$
$$= \underline{30\angle 53.13^\circ}$$

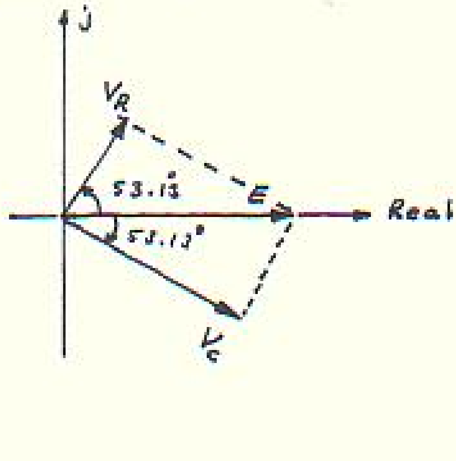
$\bar{V}_c$?

$$\bar{V}_c = \bar{I}\bar{X}_c = (5\angle 53.13^\circ)(8\angle -90^\circ)$$
$$= 40\angle -36.87^\circ$$

* you can find that:

$$\bar{E} = \bar{V}_R + \bar{V}_c$$

using the above values.



* Similarly, as in the case of series RL circuit, the active (average or true) power, reactive power can be determined.

* The active power P is

$$\boxed{P = VI\cos\phi}$$

$P = I^2 R$

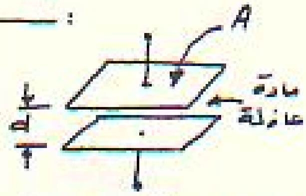
* The reactive power Q is:

$Q = I^2 X_c$

$$\boxed{Q = VI\sin\phi}$$

* and the apparent power $S = \sqrt{P^2 + Q^2}$

* Dielectric Loss and the Power Factor of a Capacitor:



For parallel-plact capacitor

$$C = \epsilon \frac{A}{d}$$

C = capacitance (F)
ϵ = dielect constant
A = Area of plate m^2
d = Separation m

* A pure (ideal) capacitor is one in which there are no losses and whose current lead the voltage by 90° as shown:

* In practice, it is impossible to get such a capacitor although close approximation is achieved by proper design.

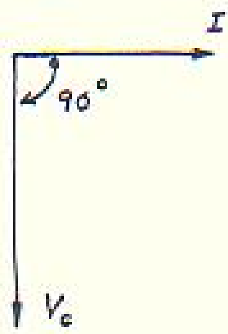
* In every capacitor, there is always some dielectric loss, and hence absorbs part of the power from the circuit. Due to this loss, the phase angle is somewhat less than 90°.

* This dielectric loss appears as heat

* ψ is the phase difference given by:

$$\psi = 90^\circ - \phi$$

where ϕ is the actual phase angle.

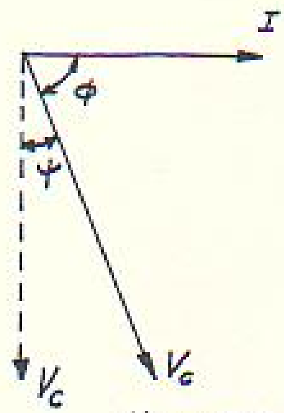


V_c and I for ideal (pure) capacitor

* Since ψ is generally small $\Rightarrow \sin\psi = \psi$

$$\therefore \boxed{\tan\psi = \psi = \cos\phi}$$

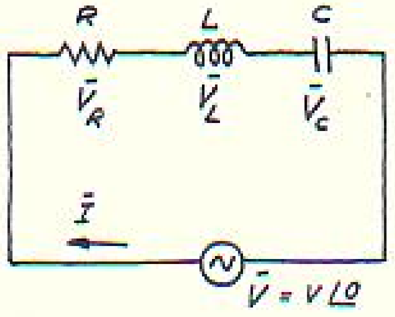
* It should be noted that dielectric loss increases with the frequency of the applied voltage.



V_c and I for Actual capacitor.

7.2.3 AC Through RLC series circuit :

$$\bar{Z} = R + jX_L - jX_c$$
$$= R + j(X_L - X_c) = Z\angle\phi$$



$$\bar{I} = \frac{V\angle 0}{Z\angle\phi} = I\angle -\phi$$

$$\bar{V}_R = \bar{I}R = (I\angle\phi)(R\angle 0^\circ)$$
$$= IR\angle -\phi$$

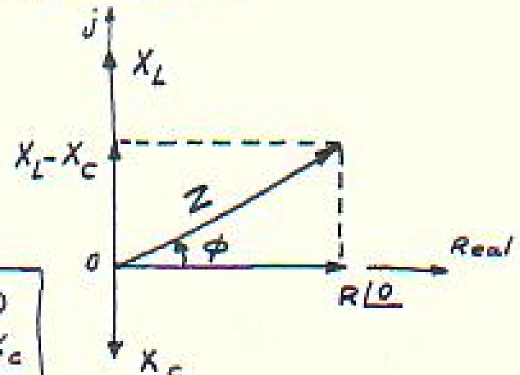
$X_L = \omega L$

$X_C = \dfrac{1}{\omega C}$

$$\bar{V}_L = \bar{I}\bar{X}_L = (I\angle -\phi)(X_L\angle 90^\circ) = IX_L\angle 90^\circ - \phi$$
$$V_c = \bar{I}X_c = (I\angle -\phi)(X_c\angle -90^\circ) = IX_c\angle -(90^\circ + \phi)$$

* The impedance diagram :

$$\phi = \tan^{-1}\frac{X_L - X_c}{R}$$

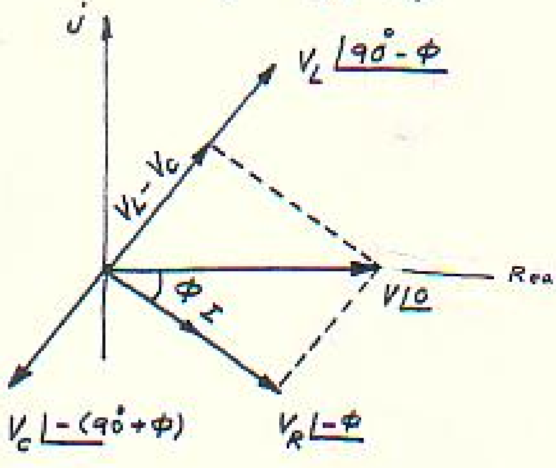


The angle ϕ may be (+ve) or (-ve) depending on the values of X_L and X_c

impedance phasor diagram for $X_L > X_c$

* The voltage phasor diagram

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_c$$
$$\therefore \bar{V} = \bar{I}R + \bar{I}\bar{X}_L + \bar{I}X_c$$

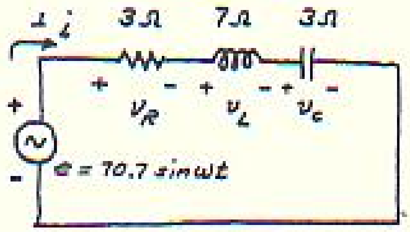


* The active, reactive and the apparent powers can be determined as mentioned previously.
* When $X_L > X_c \Rightarrow X_L - X_L = (+ve) \Rightarrow \phi =$ positive in the impedance diagram.
* When $X_L < X_c \Rightarrow X_L - X_c = (-ve)$ $\hookrightarrow \phi = (-ve)$ in the voltage phasor diagram.
  so $\phi \Rightarrow (-ve)$ in the impedance diagram, $\phi = (+ve)$ in the phasor diagram.

Example EE7

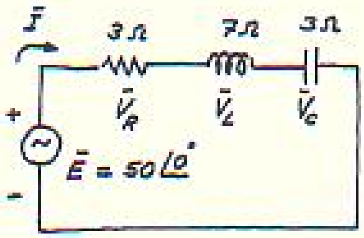
: For the circuit shown, determine:

- $\bar{Z}_T$, and draw the impedance diagram.
- $\bar{I}$, $\bar{V}_R$, $\bar{V}_L$, $\bar{V}_c$ in the phasor domain, and draw the phasor diagram.
- i, v_R, v_L, v_c in the time domain.
- The power factor of the circuit.
- The active, reactive and the apparent powers.



Solution

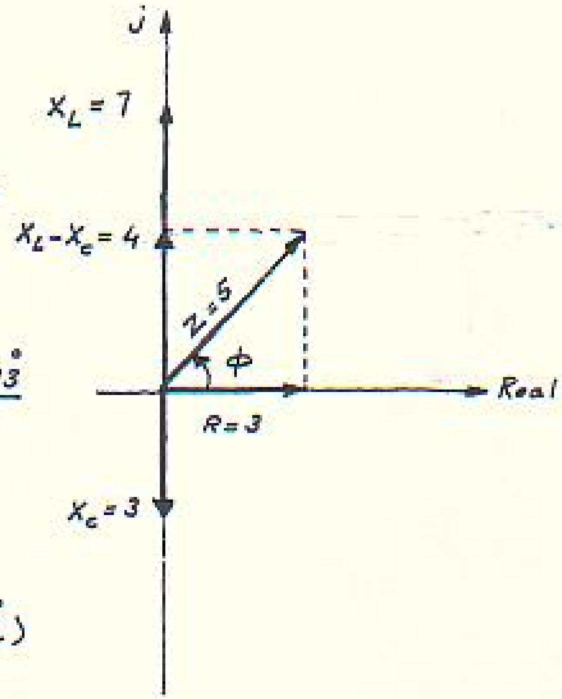
: In phasor notation the circuit is redrawn as:



* $$\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 + \bar{Z}_3$$
$$= R\angle 0 + X_L\angle 90^\circ + X_c\angle -90^\circ$$
$$= 3 + j7 - j3$$
$$= 3 + j4$$

$$\therefore \bar{Z}_T = 5\angle 53.13^\circ$$

* The impedance diagram is ⇒



$$\bar{I} = \frac{\bar{E}}{\bar{Z}} = \frac{50\angle 0}{5\angle 53.13^\circ} = 10\angle -53.13^\circ$$

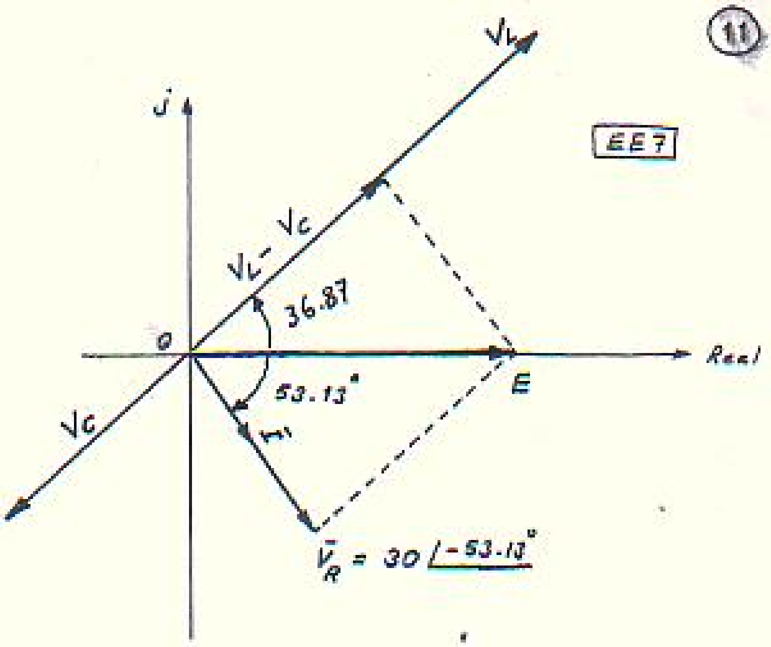
$$\bar{V}_R = \bar{I}R = (10\angle -53.13^\circ)(3\angle 0^\circ)$$
$$= 30\angle -53.13$$

$$V_L = \bar{I}\bar{X}_L = (10\angle -53.13)(7\angle 90^\circ)$$
$$= 70\angle 36.87^\circ$$

$$V_c = \bar{I}\bar{X}_c = (10\angle -53.13^\circ)(3\angle -90)$$
$$= 30\angle -143.13^\circ$$

EE7

* The phasor diagram



* The time domain

$$i = \sqrt{2}\,(10)\sin(\omega t - 53.13^\circ) = 14.14\sin(\omega t - 53.13^\circ)$$
$$v_R = \sqrt{2}\,(30)\sin(\omega t - 53.13^\circ) = 42.42\sin(\omega t - 53.13)$$
$$v_L = \sqrt{2}\,(70)\sin(\omega t + 36.87^\circ) = 98.98\sin(\omega t + 36.87^\circ)$$
$$v_c = \sqrt{2}\,(30)\sin(\omega t - 143.13^\circ) = 42.42\sin(\omega t - 143.13^\circ)$$

* The total power

$$P_T = VI\cos\phi = (50)(10)\cos 53.13^\circ = \underline{300 \text{ W}}$$

$\boxed{\text{or}}$ $P_T = I^2 R = (10)^2(3) = \underline{300 \text{ W}}$

from the voltage phasor diagram ⟺ * The power factor :

$$p.f = \cos\phi = \cos 53.13^\circ = 0.6 \ \underline{\text{lagging}}$$

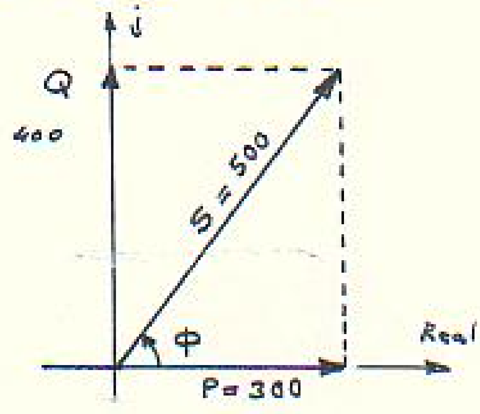
$$\text{or } p.f = \cos\phi = \frac{R}{Z_T} = \frac{3}{5} = 0.6 \text{ lagging}$$

P ⇒ * Active power = true power = $VI\cos\phi = (50)(10)\cos 53.13^\circ = 300$ W
Q ⇒ Reactive power = $Q = VI\sin\phi = (50)(10)\sin 53.13^\circ = 400$ VAR
S ⇒ Apparent power = $S = \sqrt{P^2 + Q^2} = \sqrt{(300)^2 + (400)^2} = 500$ VA

∴ The power triangle

$$\boxed{\bar{S} = P + jQ}$$

$\bar{S}$ ⇒ Complex apparent power.



7.2 Parallel AC Circuits

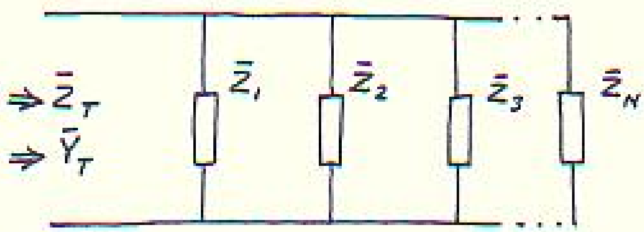
* Admittance and Susceptance

- In the dc circuit analysis, we had used the term conductance to represent the reciprocal of the resistance R; ie:

$$G = \frac{1}{R} \qquad \text{where } G \text{ is the conductance}$$

The total conductance of the paralled circuit is then found by adding the conductance of eachb branch.

- In AC circuit analycis, we define the admittance $(\bar{Y})$ as equal to $1/\bar{Z}$. For the parallel circuit shown:



* The total admittance Y_T:

$$\bar{Y}_T = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3 + \dots \bar{Y}_N$$

then since $\bar{Y} = \frac{1}{\bar{Z}}$; so the total impedance $\bar{Z}_T$:

$$\frac{1}{Z_T} = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \frac{1}{\bar{Z}_3} + \dots \frac{1}{\bar{Z}_N}$$

- As mentioned earlier, for 2 branches parallel ac circuit, then:

$$\bar{Z}_T = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$$

- Also for 3 parallel branches;

$$\bar{Z}_T = \frac{\bar{Z}_1 \bar{Z}_2 \bar{Z}_3}{\bar{Z}_1 \bar{Z}_2 + \bar{Z}_2 \bar{Z}_3 + \bar{Z}_1 \bar{Z}_3}$$

(*) INGENERAL, we have: $\bar{Z}_T = R \mp jX$, the

$$\bar{Y}_T = \frac{1}{R} \mp \frac{1}{jX} = \boxed{G \pm jB}$$

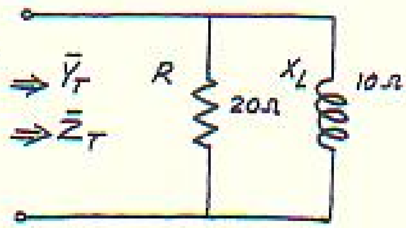
where: $G \Rightarrow$ Conductance $= \frac{1}{R}$ (Siemens, S) تقاس بوحدة

$B \Rightarrow$ Susceptance $= \frac{1}{X}$ (Siemens, S)

Example — EE7

: For the circuit shown;

a. Determine the admittance of each branch.
b. Find the input admittance.
c. Calculate the input impedance.
d. Draw the admittance diagram.



Solution:

ⓐ:

- $\bar{Y}_1 = \bar{G} = G\angle 0 = \frac{1}{R}\angle 0 = \frac{1}{20}\angle 0 = 0.05\angle 0$
$= 0.05 + j0$
$= 0.05$

- $\bar{Y}_2 = \bar{B}_L = \frac{1}{X_L}\angle -90^\circ = \frac{1}{10}\angle -90^\circ = 0.1\angle -90^\circ$
$= 0 - j0.1$
$= -j0.1$

ⓑ: $\bar{Y}_T = \bar{Y}_1 + \bar{Y}_2 = 0.05 - j0.1 = G - jB_L$

ⓒ: $\bar{Z}_T = \frac{1}{\bar{Y}_T} = \frac{1}{0.05 - j0.1} = \frac{1}{0.112\angle -63.43^\circ}$
$= 8.93\angle 63.43^\circ$

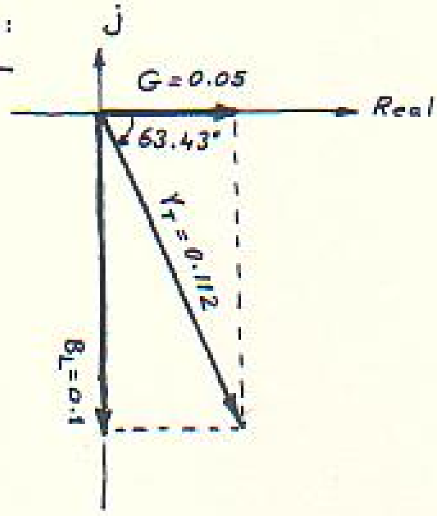
$\boxed{\text{OR}}$

$$\bar{Z}_T = \frac{\bar{Z}_1\bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} = \frac{(20\angle 0)(10\angle 90^\circ)}{20 + j10}$$

$$= \frac{200\angle 90^\circ}{22\angle 26.57^\circ}$$

$$= 8.93\angle 63.43^\circ$$ ⟵ which is the same as calculated above.

ⓓ: The admittance diagram:



EE7

Example

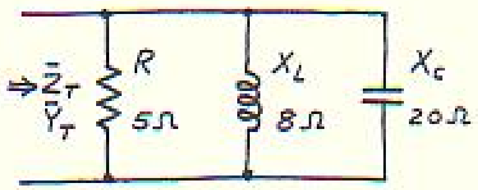
: For the circuit show;

a. Determine the admittance of each branch.

b. Find the input admittance.

c. Calculate the input impedance.

d. Draw the admittance diagram.



Solution

: ⓐ.

$$\bar{Y}_1 = \bar{G} = \frac{1}{R\angle 0} = \frac{1}{5}\angle 0 = 0.2\angle 0^\circ = 0.2 + j0 = 0.2$$

$$\bar{Y}_2 = \bar{B}_L = \frac{1}{X_L\angle 90^\circ} = \frac{1}{X_L}\angle -90^\circ = \frac{1}{8}\angle -90^\circ = 0.125\angle -90^\circ = -j0.125$$

$$\bar{Y}_3 = \bar{B}_C = \frac{1}{X_C\angle -90^\circ} = \frac{1}{X_C}\angle 90^\circ = \frac{1}{20}\angle 90^\circ = 0.05\angle 90^\circ = +j0.05$$

ⓑ.

$$\bar{Y}_T = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3$$
$$= 0.2 - j0.125 + j0.05$$
$$= 0.2 - j0.075$$
$$= 0.2136\angle -20.56^\circ$$

ⓒ.

$$\bar{Z}_T = \frac{1}{\bar{Y}_T} = \frac{1}{0.2136\angle -20.56^\circ}$$
$$= 4.68\angle 20.56^\circ$$

$\boxed{\text{or}}$

$$\bar{Z}_T = \frac{\bar{Z}_1\bar{Z}_2\bar{Z}_3}{\bar{Z}_1\bar{Z}_2 + \bar{Z}_2\bar{Z}_3 + \bar{Z}_1 Z_3}$$

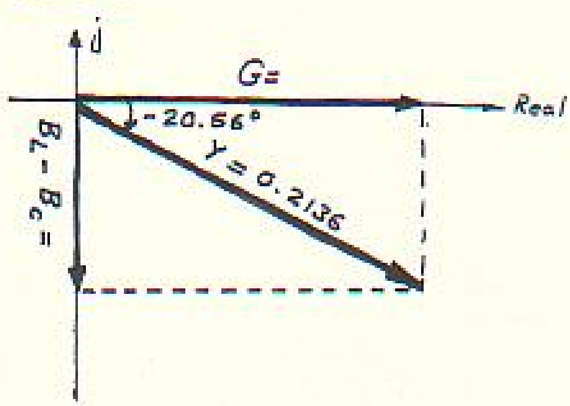
$$= \frac{(5\angle 0^\circ)(8\angle 90^\circ)(20\angle -90^\circ)}{(5\angle 0^\circ)(8\angle 90^\circ) + (8\angle 90^\circ)(20\angle -90^\circ) + (5\angle 0^\circ)(20\angle -90^\circ)}$$

$$= \frac{800\angle 0^\circ}{40\angle 90^\circ + 160\angle 0^\circ + 100\angle -90^\circ}$$

$$= \frac{800\angle 0^\circ}{j40 + 160 - j100} = \frac{800\angle 0^\circ}{160 - j60}$$

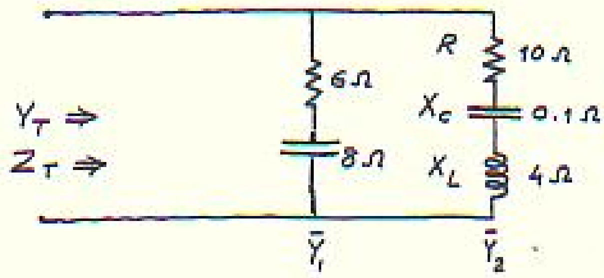
$$= \frac{800\angle 0^\circ}{170.88\angle -20.56^\circ} = \underline{\underline{4.68\angle 20.56^\circ}}$$

ⓓ. The Admittance Diagram



EE7

Example: Find the admittance of the circuit shown



Solution: $\bar{Z}_1 = 6 - j8$

$$\Rightarrow \bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{6-j8} = \frac{6+j8}{6^2+8^2} = \frac{6}{100} + j\frac{8}{100}$$

$$= 0.06 + j0.08$$

$$\bar{Z}_2 = 10 + j4 - j0.1$$
$$= 10 + j3.9$$

$$\Rightarrow \bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{10+j3.9} = \frac{10-j3.9}{10^2+3.9^2} = \frac{10}{115.21} - j\frac{3.9}{115.21}$$

$$= 0.087 - j0.034$$

$$\therefore \bar{Y}_T = \bar{Y}_1 + \bar{Y}_2$$
$$= 0.06 + j0.08 + 0.087 - j0.034$$
$$= 0.147 + j0.046$$
$$= 0.154 \angle 17.3762^\circ$$

* $\Rightarrow$ OR you can try again to get $\bar{Y}_T$ as follows:

$$\bar{Z}_T = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$$

$$= \frac{(6-j8)(10+j3.9)}{(6-j8)+(10+j3.9)}$$

and proceed to get $\bar{Z}_T \Rightarrow Y_T$ must be the same value

$$\bar{Y}_T = \frac{1}{\bar{Z}_T} = 0.154 \angle 17.3762^\circ$$

Illustrative Examples on R-L, R-C, and R-L-C Prallel AC Circuits

EE7

* R-L parallel ac circuits

Example :

For the circuit shown ;

a. $\bar{Z}_T$

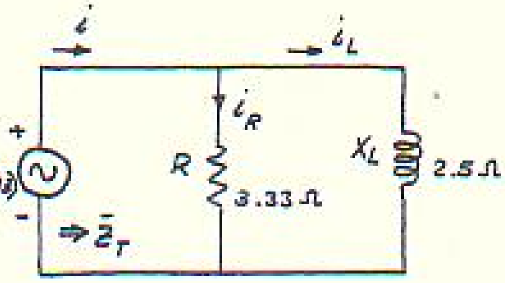
b. Draw the admittance diagram

c. The currents $\bar{I}$, $\bar{I}_R$, and $\bar{I}_L$

d. Draw the current phasor diagram.

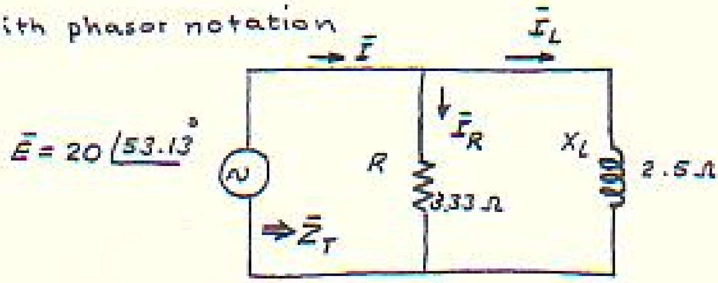
e. Calculate the active, reactive, and complex apparent powers.

f. Determine the power factor.



Solution

Draw the circuit with phasor notation



ⓐ. $\bar{Z}_T$:

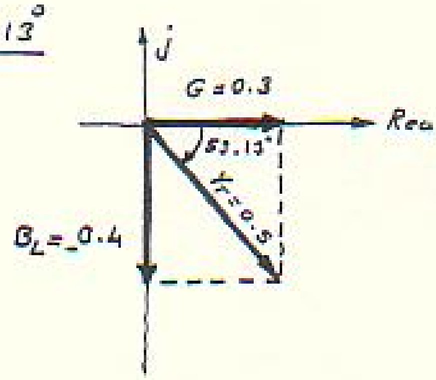
$$\bar{Y}_T = \bar{Y}_1 + \bar{Y}_2 = G + B_L = \frac{1}{3.33}\angle 0° + \frac{1}{2.5}\angle -90°$$

$$= 0.3 - j0.4$$

$$= 0.5 \angle -53.13°$$

$$\therefore \bar{Z}_T = \frac{1}{\bar{Y}_T} = \frac{1}{0.5\angle -53.13°} = 2\angle 53.13°$$

ⓑ. The admittance diagram ⇒



EE7

(c). $\bar{I} = \dfrac{\bar{E}}{\bar{Z}_T} = \dfrac{20\angle 53.13^\circ}{2\angle 53.13^\circ} = 10\angle 0^\circ$

$\bar{I}_R = \dfrac{\bar{E}}{R} = \dfrac{20\angle 53.13^\circ}{3.33\angle 0^\circ} = 6\angle 53.13^\circ$

$\bar{I}_L = \dfrac{\bar{E}}{\bar{X}_L} = \dfrac{20\angle 53.13^\circ}{2.5\angle 90^\circ} = 8\angle -36.87^\circ$

* For check

using KCL $\Rightarrow \bar{I} = \bar{I}_R + I_L$

$\Rightarrow \bar{I} = 6\angle 53.13^\circ + 8\angle -36.87$

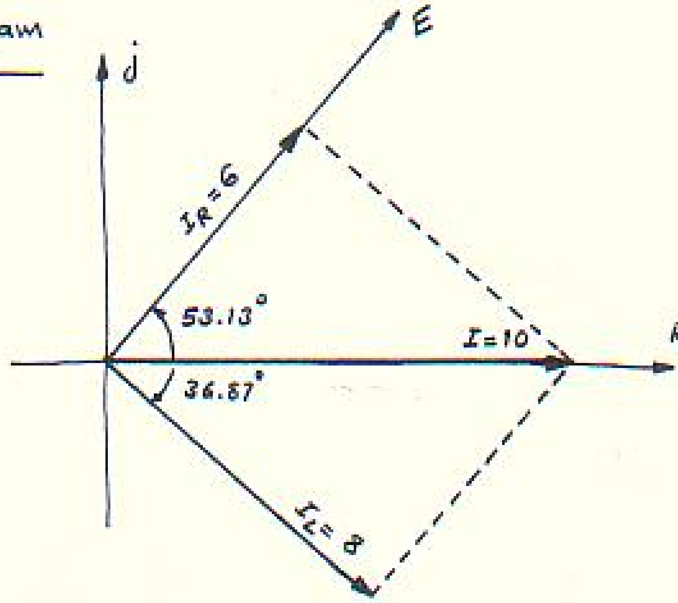
$= (3.6 + j4.8) + (6.4 - j4.8)$

$= 10 + j0$

$= \underline{\underline{10\angle 0^\circ}}$

which is the same as calculated above

(d). The current phasor diagram



(e). Active power $= P = EI\cos\phi = (20)(10)\cos 53.13^\circ$

$= 120$ W

Reactive power $= Q = EI\sin\phi = (20)(10)\sin 53.13^\circ$

$= 160$ VAR

$\therefore$ Complex apparent power $= \bar{S} = P + jQ = 120 + j160$

$\therefore \quad \bar{S} = P^2 + Q^2 = 200$ VA

from the phasor dig.

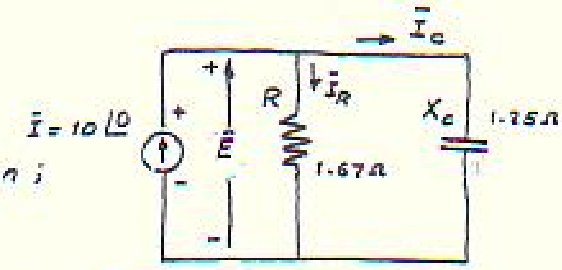
(f). The power factor $p.f = \cos\phi = \cos 53.13^\circ = 0.6$ lagging

or $\cos\phi = \dfrac{P}{EI} = \dfrac{E^2/R}{EI} = \dfrac{EG}{I} = \dfrac{G}{Y_T}$

ملاحظة : يمكن حساب $\bar{Z}_T$ حسب العلاقة ⇐ $\dfrac{\bar{Z}_1 Z_2}{\bar{Z}_1 + \bar{Z}_2}$ والحصول على النتيجة نفسها .

EE7

* R-C parallel ac circuit



Example: For the circuit shown;

a. Determine $\bar{Z}_T$.

b. Draw the admittance diagram.

c. Calculate $\bar{E}$, I_R, and $\bar{I}_c$, and draw the current phasor diagram.

d. Active, reactive, and apparent powers.

e. Determine the power factor for the circuit.

Solution: ⓐ $\bar{Y}_T = \bar{Y}_1 + \bar{Y}_2 = \frac{1}{R\angle 0} + \frac{1}{X_c\angle -90^\circ} = \frac{1}{1.67}\angle 0^\circ + \frac{1}{1.25}\angle 90$

$= 0.6 + j0.8 = 1\angle 53.13^\circ$

Also: $\bar{Z}_T = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$

$\Rightarrow \bar{Z}_T = \frac{1}{Y_T} = \frac{1}{1\angle 53.13^\circ} = 1\angle -53.13^\circ$

ⓑ.

$\bar{E} = \frac{\bar{I}}{Y_T} = \frac{10\angle 0^\circ}{1\angle 53.13^\circ}$ or $\bar{E} = \bar{I}\bar{Z}_T$

$= 10\angle -53.13^\circ$

$\bar{I}_R = \bar{E}\bar{G}$ or $\bar{I}_R = \frac{\bar{E}}{R}$

$= (10\angle -53.13^\circ)(0.6)$

$= 6\angle -53.13^\circ$

$I_c = \bar{E}\bar{B}_c$ or $\bar{I}_c = \frac{\bar{E}}{\bar{X}_c}$

$= (10\angle -53.13^\circ)(0.8\angle 90)$

$= 8\angle 36.87^\circ$

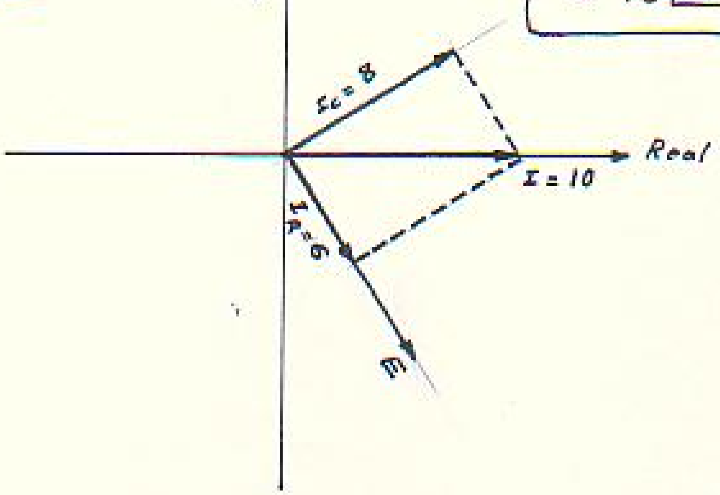
$\Rightarrow$ As a check?

$\bar{I} = \bar{I}_R + \bar{I}_c$

$= 6\angle -53.13 + 8\angle 36.87^\circ$

$= 10\angle 0^\circ$

ⓒ. The phasor diagram:



d. Active power $= P = EI\cos\phi$

$= (10)(10)\cos 53.13^\circ$

$= 60\ W$

$*$ or $P = E^2 G$

$= (10)^2(0.6)$

$= 60\ W$

Reactive power $= Q = EI\sin\phi$

$= (10)(10)\sin 53.13^\circ$

$= 80\ VAR$

$\therefore$ Apparent power $S = \sqrt{P^2 + Q^2} = \sqrt{(60)^2 + (80)^2}$

$= 100\ VA$

e. The power factor

$p.f = \cos 53.13$

$= 0.6$ <u>leading</u> ⇐ from the phasor diagram $\bar{I}$ leads $\bar{E}$

* R-L-C parallel ac circuit:

Example: For the circuit shown;

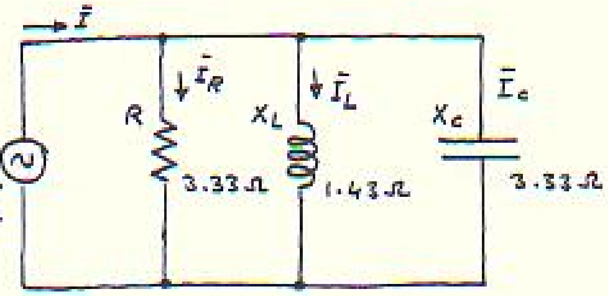
a. Determine $\bar{Z}_T$.

b. Calculate $\bar{I}, \bar{I}_R, \bar{I}_L$ & $\bar{I}_c$

c. Draw the phasor diag. $\bar{E} = 100\angle 53.13^\circ$

d. Calculate the active (real) power

e. Determine the power factor



Solution: a. $\bar{Z}_T = ?$ ⇒

$$\bar{Y}_T = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3$$

$$= \frac{1}{R}\angle 0^\circ + \frac{1}{X_L}\angle -90^\circ + \frac{1}{X_c}\angle 90^\circ$$

$$= \frac{1}{3.33}\angle 0^\circ + \frac{1}{1.43}\angle -90^\circ + \frac{1}{3.33}\angle 90^\circ$$

$$= 0.3 - j0.7 + j0.3$$

$$= 0.3 - j0.4$$

$$= 0.5\angle -53.13^\circ$$

$$\therefore \bar{Z}_T = \frac{1}{\bar{Y}_T} = \frac{1}{0.5\angle -53.13^\circ} = 2\angle 53.13^\circ$$

b. $$\bar{I} = \frac{\bar{E}}{\bar{Z}_T} = \frac{100\angle 53.13^\circ}{2\angle 53.13^\circ} = 50\angle 0^\circ$$

$\bar{I}_R = \frac{\bar{E}}{R} = \checkmark$ ممكن ⇐

$$\bar{I}_R = \bar{E}\bar{G} = (100\angle 53.13^\circ)(0.3\angle 0^\circ)$$

$$= 30\angle 53.13^\circ$$

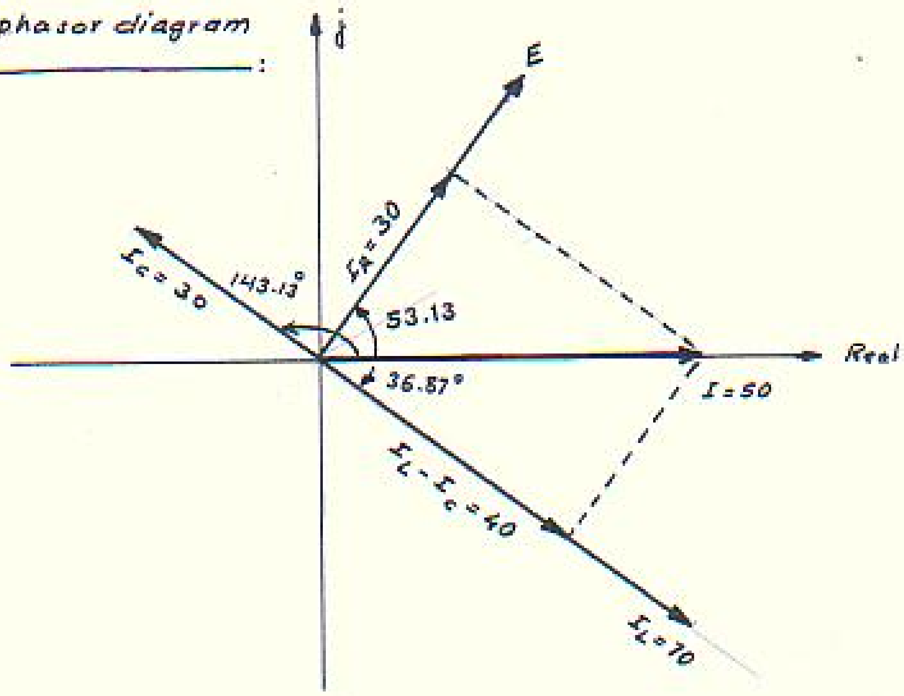
$\Rightarrow \bar{I}_L = \dfrac{\bar{E}}{\bar{X}_L}$ $\boxed{EE7}$

$$\bar{I}_L = \bar{E}\bar{B}_L = (100\angle 53.13^\circ)(0.7\angle -90) = 70\angle -36.87^\circ$$

$$I_c = \bar{E}\bar{B}_c = (100\angle 53.13^\circ)(0.3\angle 90) = 30\angle 143.13^\circ$$

$\boxed{\text{Prove that : } \bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_c}$

ⓒ. The current phasor diagram :



ⓓ. Active power (Real power) $= P$

$$= EI\cos\phi$$
$$= (100)(50)\cos 53.13^\circ$$
$$= 3000 \text{ W}$$
$$= 3.0 \text{ kW}$$

$\Rightarrow$ or $P = E^2 G$

$$= (100^2)(0.3)$$
$$= 3.0 \text{ kW}$$

ⓔ. The power factor $\Rightarrow$ $P.f = \cos\phi$

$$= \cos 53.13$$
$$= 0.6 \text{ lagging}$$

$\Leftarrow$ from the phasor diagram.

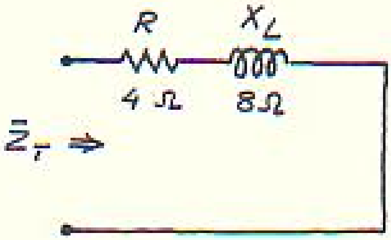
$$\bar{Z}_T = \frac{\bar{Z}_1\bar{Z}_2\bar{Z}_3}{\bar{Z}_1\bar{Z}_2 + \bar{Z}_2\bar{Z}_3 + \bar{Z}_1\bar{Z}_3}$$

$\Leftarrow$ ملاحظة : من الممكن كذلك حساب Z_T والحصول على النتيجة نفسها .

Tutorial Sheet Nº 7

TS7

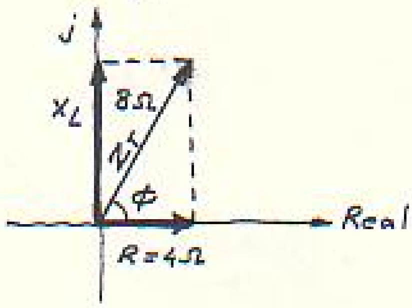
Example: Draw the impedance diagram for the circuit shown and find the total impedance.



Solution: $\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2$

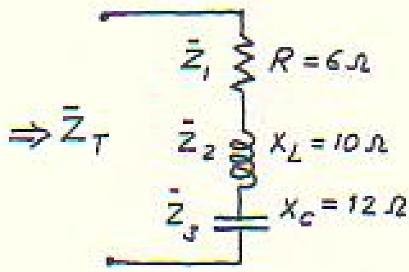
$$= R\angle 0 + X_L \angle 90^\circ = R + jX_L = 4 + j8$$

$$\therefore \bar{Z}_T = 8.944 \angle 63.43^\circ \ \Omega$$



Example: Determine the input impedance to the series network shown

Solution:



$$\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 + \bar{Z}_3$$

$$= R\angle 0 + X_L \angle 90 + X_C \angle -90$$

$$= R + jX_L - jX_C$$

$$= R + j(X_L - X_C)$$

$$= 6 + j(10 - 12)$$

$$= 6 - j2$$

$$\Rightarrow \bar{Z}_T = 6.325 \angle -18.43^\circ \ \Omega$$

TS7

Example

: A 60 Hz sinusoidal voltage ($v = 141 \sin\omega t$) is applied to a series R-L circuit. The values of the resistance and the inductance are $3\,\Omega$ and 0.0106 H respectively.

(a). Compute the rms value of the current in the circuit and its phase angle with respect to the voltage.

(b). Write the expression for the instantaneous current in the circuit.

(c). Find the average power dissipated by the circuit.

(d). Calculate the p.f of the circuit.

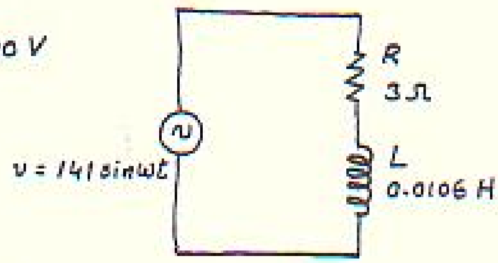
Solution

: We have ;

$$v = V_m \sin\omega t$$

$$\Rightarrow V = \frac{V_m}{\sqrt{2}} = \frac{141}{\sqrt{2}} = 100\ V$$

$$\therefore \bar{V} = 100\angle 0$$



(a). $\bar{I} = \dfrac{\bar{V}}{\bar{Z}}$

$$\begin{aligned}\bar{Z} &= R + jX_L \\ &= R + j(2\pi f L) \\ &= 3 + j(2\pi \times 60 \times 0.0106) \\ &= 3 + j4 \\ \therefore \bar{Z} &= 5\angle 53.1^\circ\end{aligned}$$

$$\Rightarrow \bar{I} = \frac{100\angle 0^\circ}{20\angle 53.1^\circ}$$

$$\therefore \bar{I} = 20\angle -53.1$$

$\Rightarrow$ the current lags the voltage by 53.1°

(b). $i = I_m \sin(\omega t - 53.1^\circ)$

$$= \sqrt{2}(20)\sin(\omega t - 53.1^\circ = \underline{28.28 \sin(\omega t - 53.1^\circ)}$$

(c). $P = VI\cos\phi$

$$= (100)(20)\cos 53.1^\circ = 1200\ W$$

or $P = I^2 R = (20)^2(3) = 1200\ W$

(d). $p.f = \cos\phi$

$$= \cos 53.1^\circ$$

$$= 0.6 \quad \underline{\text{lagging}}$$

Example

: A two elements series circuit is connected across an ac circuit having a source ($e = \sqrt{2}(200)\sin(\omega t + 20^\circ)$ V. The current in the circuit is then found to be $i = \sqrt{2}(10)\cos(341t - 25^\circ)$. Determine the parameter's of the circuit.

TS1

Solution:

The applied voltage is :

$$v = \sqrt{2}(200)\sin(\omega t + 20^\circ)$$

$$\Rightarrow \bar{V} = 200\angle 20^\circ$$

The current is :

$$i = \sqrt{2}(10)\cos(\omega t - 25^\circ)$$
$$= \sqrt{2}(10)\sin(\omega t - 25^\circ - 90^\circ)$$
$$\therefore\ i = \sqrt{2}(10)\sin(\omega t + 65^\circ)$$

$$\Rightarrow \bar{I} = 10\angle 65^\circ$$

$$\Rightarrow \bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{200\angle 20^\circ}{10\angle 65^\circ} = 20\angle -65^\circ$$
$$= 14.14 - j\,14.14$$

Note $\phi = -45^\circ$ (leading)

This impedance represents a series circuit with $R = 14.14\ \Omega$ and a capacitive reactance (because of the $-j$) of $X_c = 14.14\ \Omega$

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

* من معادلة التيار :
$\omega = 314$ rad/sec.

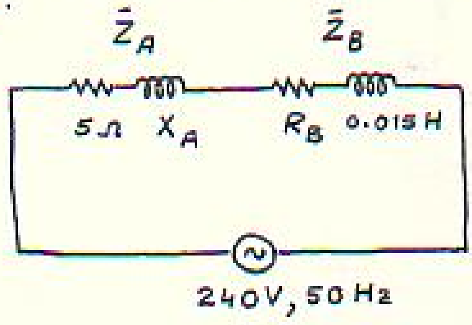
$$\therefore\ 14.14 = \frac{1}{314C} \Rightarrow C = \frac{1}{(14.14)(314)} = 2.25\times10^{-4}\ F$$

$\therefore$ The circuit has $R = 14.14\ \Omega$
and $C = 225\ \mu F$

Example: Two coils A and B are connected in series across 240 V, 50 Hz supply. The resistance of A is 5 Ω and the inductance of B is 0.015 H. If the input supply is 3 kW and 2 kVAR, find the resistance of B and the inductance of A. Calculate the voltage across each coil.

Solution:

* From the power triangle, and the circuit shown,



$$S = \sqrt{P^2 + Q^2} = \sqrt{3^2 + 2^2}$$
$$= 3.606\ KVA$$

$$S = VI \Rightarrow I = \frac{S}{V} = \frac{3606}{240} = 15.025\ A$$

But, $P = 3kW = 300\ W$
$= I^2 R_T = I^2(R_A + R_B)$

$$\therefore\ 3000 = (15.025)^2(R_A + R_B)$$

$$\Rightarrow R_A + R_B = 13.3\ \Omega$$

$\therefore$ Since $R_A = 5\ \Omega \Rightarrow R_B = 13.3 - 5 = \underline{8.3\ \Omega}$

TS7

Similarly, we have:

ملاحظة : من الممكن حل هذا المثال
بأكثر من طريقة، والوصول إلى
النتائج نفسها.

$$Q = 2\ kVAR = 2000\ VAR$$
$$= I^2 X_{L_T} = (15.03)^2 X_{L_T}$$

$$\therefore X_{L_T} = \frac{2000}{(15.03)^2} = 8.85\ \Omega$$

$$X_{L_T} = X_A + X_B \Rightarrow X_A = 8.85 - X_B$$
$$= 8.85 - (2\pi f L_B)$$
$$= 8.85 - (2\pi \times 50 \times 0.015)$$
$$= 8.85 - 4.713 = 4.13\ \Omega$$

$$\therefore \bar{Z}_A = R_A + jX_A = 5 + j\,4.13 = 6.48\,\underline{|39.57^\circ}$$

$$\text{and } \bar{Z}_B = R_B + jX_B = 8.3 + j\,4.713 = 9.54\,\underline{|29.59^\circ}$$

$X_B = 2\pi f L_B$
$= 2\pi \times 50 \times 0.015$

$$\therefore \bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 = 5 + j4.13 + 8.3 + j4.713$$
$$=$$

$$\therefore \bar{V}_A = \bar{I}\bar{Z}_A = \checkmark$$

$$\bar{V}_B = \bar{I}\bar{Z}_B = \checkmark$$

$$\text{or} \quad \bar{V}_A = \frac{\bar{V}\bar{Z}_A}{\bar{Z}_A + \bar{Z}_B} = \checkmark$$

The results must be the same.

and

$$\bar{V}_B = \frac{\bar{V}\bar{Z}_B}{\bar{Z}_A + \bar{Z}_B} = \checkmark$$

Example:

A 240 V, 50 Hz series R-C circuit takes an rms current of 20 A. The maximum value for the current occurs 1/900 seconds before the maximum value of the voltage. Calculate:

(a). The power factor.

(b). Average power.

(c). The parameters of the circuit.

Solution:

The time duration of the voltage $(T) = 0.02 = 0.02$ sec.

$$\therefore 0.05 \text{ sec.} \Rightarrow 360^\circ, \text{ then}$$

$$(1/900) \text{ sec} \Rightarrow \left[\frac{360^\circ (1/900)}{0.02}\right]^\circ$$ = The phase shift angle (ϕ)

$$\therefore \phi = 20^\circ$$

TS7

(a). $\therefore$ $p.f = \cos\phi = \cos 20^\circ$

$= 0.9397$ (leading)

(b). Average power $= P = VI\cos\phi$

$= 240(20)\cos 20^\circ$

$= 4510$ W

$= 4.510$ kW

(c). $\bar{Z} = \dfrac{\bar{V}}{\bar{I}} = \dfrac{240\angle 0^\circ}{20\angle 20^\circ} = 12\angle -20^\circ$

$= 20\cos(-20) + j\,12\sin(-20)$

$= 11.28 - j\,4.1$

$\therefore$ $\bar{Z}$ is composed of $R = 11.28\ \Omega$, and

$X_c = 4.1\ \Omega$

$X_c = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

$\therefore C = \dfrac{1}{2\pi f X_c} = 7.76\times 10^{-4}$ F

$= 776\ \mu F$

Example

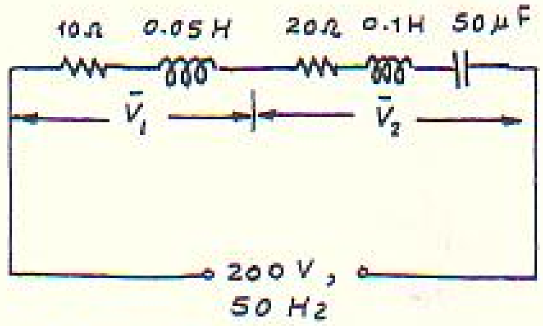
: Draw the phasor diagram for the circuit shown, indicating the resistance and the reactance drop, the terminal voltages $\bar{V}_1$ and $\bar{V}_2$ and the current. Find the values of:

(a). The current $\bar{I}$.

(b). $\bar{V}_1$

(c). $\bar{V}_2$

(d). The power factor



Solution

:

$R_T = 10 + 20 = 30\ \Omega$

$L_T = 0.05 + 0.1 = 0.15$ H

$\Rightarrow X_L = \omega L = 2\pi f L_T = 2\pi(50)(0.15)$

$= 47.1\ \Omega$

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi(50)(50\times 10^{-6})} = 63.7\ \Omega$

$\therefore \bar{Z}_T = \sqrt{R^2 + (X_L - X_c)^2} = \sqrt{(30)^2 + (47.1 - 63.7)^2}$

$= 34.3\angle -28.96^\circ$

(a). $\therefore \bar{I} = \dfrac{\bar{V}}{\bar{Z}} = \dfrac{200\angle 0^\circ}{34.3\angle -28.96} = 5.83\angle 28.96$ leading

TST

(b). $\bar{V}_1 = ?$

$$\bar{V}_1 = \bar{I}\,\bar{Z}_1 \quad \Rightarrow \quad \bar{Z}_1 = 10 + jX_{L_1}$$

$$= 10 + j\,(2\pi f L_1)$$

$$= 10 + j\,(2\pi (50)\,0.05)$$

$$= 10 + j\,15.7$$

$$\therefore \bar{Z}_1 = 18.6 \underline{|57.5}$$

$$\therefore \bar{V}_1 = (5.83 \underline{|28.96})(18.6 \underline{|57.5})$$

$$= 108.4 \underline{|86.46^\circ}$$

(c). $\bar{V}_2 = ?$

$$\bar{V}_2 = \bar{I}\,\bar{Z}_2 \quad \Rightarrow \quad \bar{Z}_2 = 20 + jX_{L_2} - jX_{C_2}$$

$$= 20 + j\,(2\pi f L_2) - \frac{1}{2\pi f C_2}$$

$$= 20 + j\,31.4 - j\,63.7$$

$$= 20 - j\,32.3$$

$$\therefore \bar{Z}_2 = 37.74 \underline{|-58.2^\circ}$$

$$\therefore \bar{V}_2 = \bar{I}\,\bar{Z}_2 = (5.83 \underline{|28.96^\circ})(37.74 \underline{|-58.2^\circ})$$

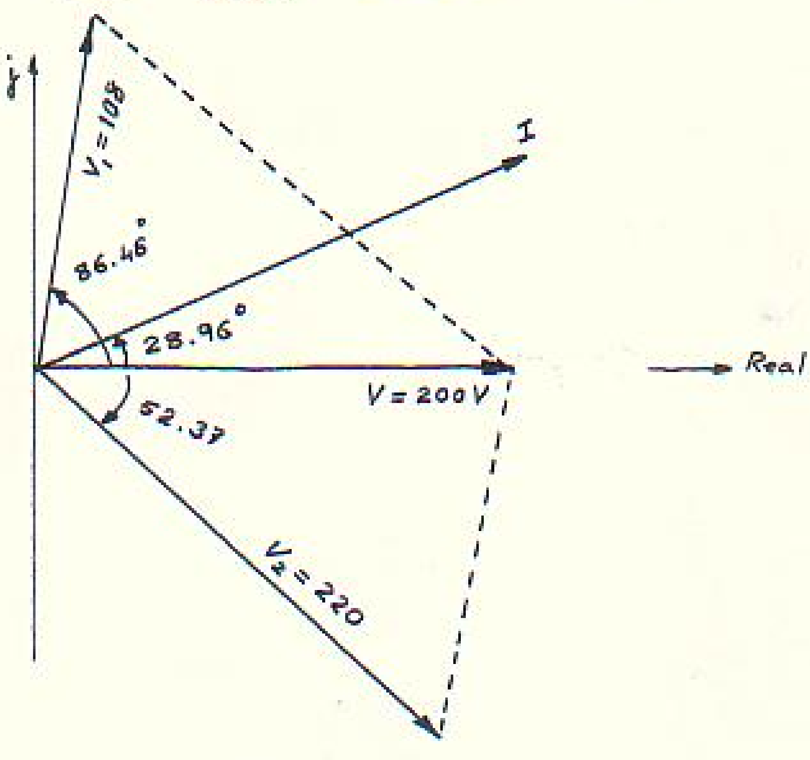
$$= 220.1 \underline{|-52.37}$$

(d). The combined (overall) power factor of the circuit:

- from part (a) $\Rightarrow$ $p.f = \cos\phi = \cos 28.96^\circ = 0.87$ <u>leading</u>

- <u>or</u> $p.f = \dfrac{R}{Z_T} = \dfrac{30}{34.3} = \underline{0.87}$ (leading)

<u>The phasor diagram</u>



TS7

Example

: In a circuit it is found that the applied voltage is to lag the current by 30°.

(a). Is the power factor lagging or leading ?
(b). What is the value of the power factor ?
(c). Is the circuit inductive or capacitive ?

Solution

:

(a). The power factor is leading, since the current leads the voltage.

(b). The power factor is $p.f = \cos\phi$

$$= \cos 30$$

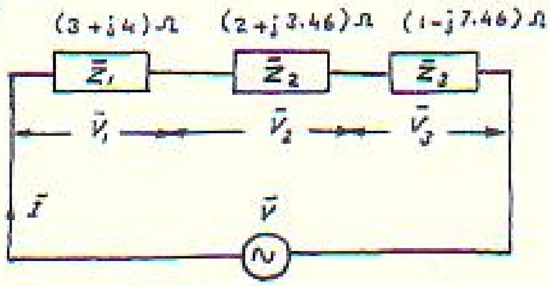
$$= 0.866 \text{ (lead)}$$

(c) The circuit is capacitive.

Example

: In the circuit diagram of the Fig. shown, the voltage drop across $\bar{Z}_1$ is $(10 + j0)$ volts. Find out :

(a). The current in the circuit.
(b). The voltage drop across $\bar{Z}_2$ and $\bar{Z}_3$
(c). The voltage of the source.



Solution

: (a). $\bar{I} = \dfrac{\bar{V}}{\bar{Z}_T}$ $\Rightarrow$ $\bar{Z}_T = ✓$, $\bar{V} = ?$

$$\text{or } I = \frac{\bar{V}_1}{\bar{Z}_1} = \frac{10+j0}{3+j4} = \frac{10\angle 0^\circ}{5\angle 53.1^\circ} = 2\angle -53.1^\circ$$

$$= 2(\cos 53.1^\circ - j\sin 53.1^\circ) = 1.2 - j1.6$$

(b). $\bar{V}_2 = \bar{I}\bar{Z}_2 = (1.2-j1.6)(2+j3.46) = 7.936+j0.952$ volts

$\bar{V}_3 = \bar{I}\bar{Z}_3 = (1.2-j1.6)(1-j7.46) = -10.74-j10.55$ volts

(c). $\bar{V} = \bar{V}_1 + \bar{V}_2 + \bar{V}_3$

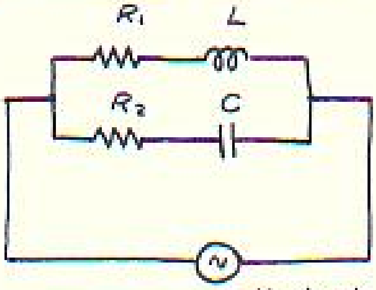
$$= (10+j0) + (7.936+j0.952) + (-10.74-j10.55)$$

$$= 7.2 - j9.6 = 12\angle -53.1^\circ \text{ volts}$$

نلاحظ ان الفولتية الكلية والتيار المار في الدائرة كلاهما بنفس الطور والسبب في ذلك ان $\bar{Z}_T$ في هذه الدائرة لا تمثل عدداً مركباً بل قيمة حقيقية فقط (real part) اي انها تمثل مقاومة فقط والتي بدورها لا تؤثر في الطور بين الفولتية والتيار.

Example TS7

: Derive an expression for the equivalent impedance for the circuit in the Fig. shown, in terms of the circuit parameters



$v = V_m \sin \omega t$

Solution :

Let $\bar{Z}_1 = R_1 + jX_L = R_1 + j\omega L$

& $\bar{Z}_2 = R_2 - jX_C = R_2 - j\dfrac{1}{\omega C}$

$$\therefore \quad \bar{Z}_T = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$$

$$= \frac{(R_1 + j\omega L)(R_2 - j\frac{1}{\omega C})}{(R_1 + j\omega L) + (R_2 - j\frac{1}{\omega C})}$$

$$= \frac{R_1 R_2 + j\omega L R_2 - j\frac{R_1}{\omega C} + \frac{L}{C}}{(R_1 + R_2) + j(\omega L - \frac{1}{\omega C})}$$

$$\Rightarrow \frac{\left(R_1 R_2 + j\omega L R_2 - j\frac{R_1}{\omega C} + \frac{L}{C}\right)(R_1 + R_2)}{(R_1 + R_2)^2 + (\omega L - \frac{1}{\omega C})^2} - j\,\frac{\left(R_1 R_2 + j\omega L R_2 - j\frac{R_1}{\omega C} + \frac{L}{C}\right)(\omega L - \frac{1}{\omega C})}{(R_1 + R_2)^2 + (\omega L - \frac{1}{\omega C})^2}$$

$$= \frac{(R_1 R_2 + \frac{L}{C})(R_1 + R_2) + (\omega L R_2 - \frac{R_1}{\omega C})(\omega L - \frac{1}{\omega C})}{(R_1 + R_2)^2 + (\omega L - \frac{1}{\omega C})^2} - j\,\frac{(R_1 + R_2)(\frac{R_1}{\omega C} - \omega L R_2) + (R_1 R_2 - \frac{L}{C})(\omega L - \frac{1}{\omega C})}{(R_1 + R_2)^2 + (\omega L - \frac{1}{\omega C})^2}$$

Simplifying further , then :

$$\bar{Z}_T = \frac{R_1 R_2 (R_1 + R_2) + \omega^2 L^2 R_2 + \frac{R_1}{\omega^2 C^2}}{(R_1 + R_2)^2 + (\omega L - \frac{1}{\omega C})^2} + j\,\frac{\omega L R_2^2 - \frac{R_1^2}{\omega C} - \frac{L}{C}(\omega - \frac{1}{\omega C})}{(R_1 + R_2)^2 + (\omega L - \frac{1}{\omega C})^2}$$

* You can also derive an expression for the admittance of the circuit Y_T , and the result will be as :

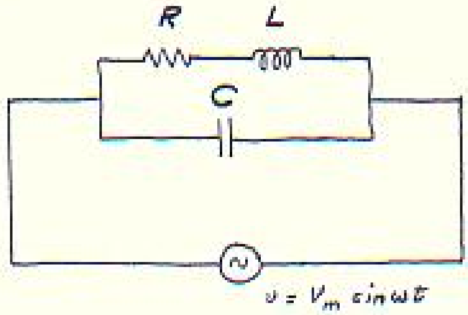
$$\bar{Y}_T = \frac{R_1 + \omega^2 C^2 R_1 R_2 (R_1 + R_2) + \omega^4 L^2 C^2 R_2^2}{(R_1 + \omega^2 L^2) + (1 + \omega^2 C^2 R_2^2)} + j\omega \left[\frac{C R_1^2 - L + \omega^2 LC (L - C R_2^2)}{(R_1 + \omega^2 L^2) + (1 + \omega^2 C^2 R_2^2)} \right]$$

ملاحظة : ربما يعتقد الطالب انه لا فكرة خاصة في هذا المثال عدا الجهد الذي يتطلبه الحل وهو ليس بالقليل . ان الفكرة من هذا المثال سوف تتضح خلال دراسة موضوع الرنين (Resonance) الذي تتطلب معظم مسائله اشتقاق تردد الرنين ومعالم اخرى من خلال اشتقاق $\bar{Z}_T$ او Y_T .

TST

Example (HW)

: Derive expressions for the equivalent impedance and (or) the admittance for the circuit shown.



Answer

:

$$\bar{Z}_T = \frac{R + j\omega\left[L(1-\omega^2 LC^2) - CR^2\right]}{(1-\omega^2 LC)^2 + \omega^2 C^2 R^2}$$

and

$$\bar{Y}_T = \frac{R - j\omega\left[L(1-\omega^2 LC) - CR^2\right]}{R^2 + \omega^2 L^2}$$

* Please check the answers.

Example

: For the circuit shown in the Fig., determine:

(a). $\bar{Z}_T$

(b). $\bar{I}_T$

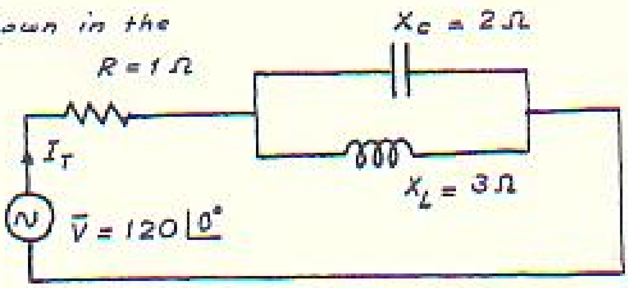
(c). I_c

(d). $\bar{V}_R$

(e). $\bar{V}_c$

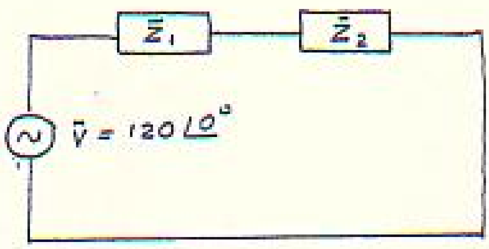
(f). Average power

(g). The power factor of the circuit.



Solution

: Redraw the circuit (option):



Let $\bar{Z}_1 = R = 1\angle 0^\circ\ \Omega$

& $Z_2 = X_c \,/\!/\, X_L$

$$= \frac{X_c X_L}{X_c + X_L} = \frac{(-j2)(j3)}{-j2 + j3}$$

$$= \frac{6\angle 0^\circ}{j1} = \frac{6\angle 0}{1\angle 90^\circ}$$

$$= 6\angle -90^\circ = -j6\ \Omega$$

(a). $\therefore\ \bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 = 1 - j6 = 6.08\angle{-80.54^\circ}\ \Omega$ $\boxed{T57}$

(b). $\bar{I}_T = \dfrac{\bar{V}}{\bar{Z}} = \dfrac{120\angle{0^\circ}}{6.08\angle{-80.54^\circ}} = 19.74\angle{80.54^\circ}$ A

(c). $I_c = ?$

Using the current divider rule, I_c can be calculated as:

$$\bar{I}_c = I_T \frac{X_L}{X_L + X_c} = 19.74\angle{80.54^\circ} \cdot \frac{3\angle{90^\circ}}{1\angle{90^\circ}}$$

$$= 59.22\angle{80.54^\circ} \text{ A}.$$

(d). $\bar{V}_R = ?$

$$\bar{V}_R = \bar{I}_T \bar{Z}_1 = (19.74\angle{80.54^\circ})(1\angle{0})$$

$$= 19.74\angle{80.54^\circ} \text{ V}$$

(e) $\bar{V}_c = ?$

$$\bar{V}_c = \bar{I}_T \bar{Z}_2 = (19.74\angle{80.54^\circ})(6\angle{-90^\circ})$$

$$= 118.44\angle{-9.46^\circ} \text{ V}$$

$\boxed{OR}$ $\bar{V}_c = \bar{I}_c \bar{X}_c = (59.22\angle{80.54^\circ})(2\angle{-90^\circ})$

$$= 118.44\angle{-9.46^\circ}$$

(f). Average power = Active power

$$P = I_T^2 R = (19.74)^2 \times 1 = 389.67 \text{ W}$$

or $P = VI\cos\phi = 389.67$ W

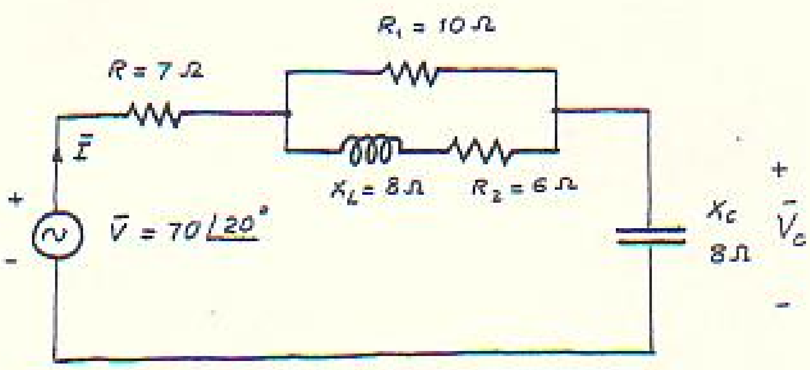
(g). Power factor $= \cos\phi$ $\qquad \phi = 80.54^\circ$

$$\therefore\ p.f = \cos 80.54^\circ$$
$$= 0.164 \ \text{\underline{leading}}$$

Example (HW):

(a). Calculate the voltage V_c using the voltage divider rule.

(b). Calculate the current I



Answer

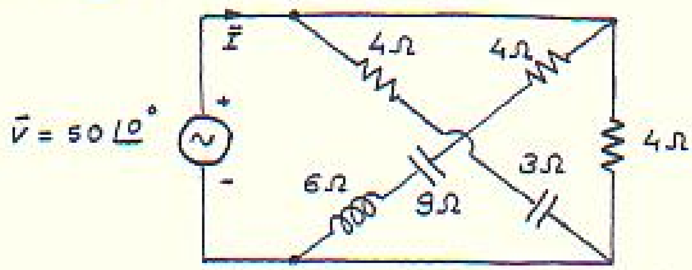
(a). $\bar{V}_c = 42.42\angle{-45.38^\circ}$ volts

(b). $\bar{I} = 5.3\angle{44.62^\circ}$ A

TS7

Example

: Find the current I in the circuit shown.



Solution

: Let $\bar{Z}_1 = 4 - j3$

$$\bar{Z}_2 = 4 + j6 - j9 = 4 - j3$$

لوحظ ان $\bar{Z}_1$, $\bar{Z}_2$, $\bar{Z}_3$ مرتبطة على التوازي

and

$$\bar{Z}_3 = 4 + j0 = 4\ \Omega$$

لوحظ ان $\bar{Z}_1 = \bar{Z}_2$

$$\bar{Z}_T = \bar{Z}_1 // Z_2 // Z_3$$

since $\bar{Z}_1 = Z_2 \Rightarrow Z_{12} = \frac{Z_1}{2} = \frac{4-j3}{2} = \underline{2 - j1.5\ \Omega}$

$$\therefore \bar{Z}_T = \bar{Z}_{12} // \bar{Z}_3 = \frac{\bar{Z}_{12}\bar{Z}_3}{\bar{Z}_{12} + \bar{Z}_3} = \frac{4(2-j1.5)}{4+2-j1.5}$$

$$\therefore \bar{Z}_T = \frac{8-j6}{6-j1.5} = \frac{10\angle -36.87^\circ}{6.18\angle -14.04^\circ} = 1.62\angle -22.83^\circ$$

$$\therefore \bar{I}_T = \frac{\bar{V}}{\bar{Z}_T} = \frac{50\angle 0^\circ}{1.62\angle -22.83^\circ} = \underline{30.86\angle 22.83^\circ}$$

Note

: Z_T can be calculated from :

$$\bar{Z}_T = \frac{\bar{Z}_1\bar{Z}_2\bar{Z}_3}{\bar{Z}_1\bar{Z}_2 + \bar{Z}_2\bar{Z}_3 + \bar{Z}_1\bar{Z}_3}$$

or $\bar{Z}_T = \frac{1}{\bar{Y}_T}$ where $\bar{Y}_T = Y_1 + Y_2 + Y_3$

$$\bar{Y}_1 = \frac{1}{\bar{Z}_1}, \quad \bar{Y}_2 = \frac{1}{\bar{Z}_2}, \quad \bar{Y}_3 = \frac{1}{\bar{Z}_3}$$

والحصول على النتيجة نفسها .

Example: For the circuit shown,

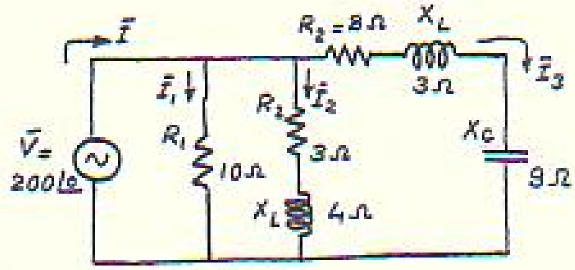
(a). Compute $\bar{I}$.

(b). Find $\bar{I}_1$, $\bar{I}_2$ and $\bar{I}_3$

(c). Verify Kirchhoff's Current law by showing that:

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$$

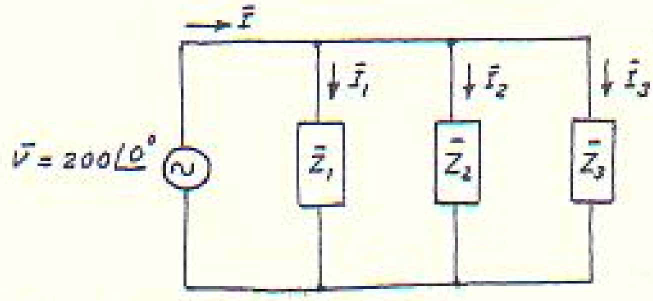
(d). Find the total impedance of the circuit.



Solution: Redraw the circuit as shown in the Fig. below;

where;

$$\bar{Z}_1 = 10\angle 0° = 10\,\Omega$$
$$\bar{Z}_2 = 3 + j4$$
$$\bar{Z}_3 = 8 + j3 - j9 = 8 - j6$$



$$\bar{Y}_T = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3 = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \frac{1}{\bar{Z}_3}$$

$$= \frac{1}{10} + \frac{1}{3+j4} + \frac{1}{8-j6}$$

$$= \left(\frac{1}{10}\right) + \frac{3}{9+16} - j\frac{4}{9+16} + \frac{8}{64+36} + j\frac{6}{64+36}$$

$$= \frac{1}{10} + \frac{3}{25} + \frac{8}{100} - j\frac{4}{25} + j\frac{6}{100}$$

$$\therefore \bar{Y}_T = 0.3 - j0.1 \quad (S)$$

(a). $\bar{I} = ?$

$$\bar{I} = \bar{V}.\bar{Y}_T = 200\angle 0° (0.3 - j0.1)$$
$$= \underline{60 - j20}$$

(b).

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{200\angle 0°}{10\angle 0°} = 20\angle 0° \quad A$$

$$I_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{200\angle 0°}{5\angle 53.13°} = 40\angle -53.13° \quad A$$

and

$$\bar{I}_3 = \frac{\bar{V}}{\bar{Z}_3} = \frac{200\angle 0°}{10\angle -36.87°} = 20\angle 36.87°$$

(c)

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + I_3$$
$$60 - j20 = 20\angle 0° + 40\angle -53.13° + 20\angle 36.87°$$
$$= (20 + j0) + (24 - j32) + (16 + j12)$$
$$\therefore \underline{60 - j20} = \underline{60 - j20}$$

TS7

(d). $\bar{Z}_T = ?$

$$\bar{Z}_T = \frac{1}{\bar{Y}_T} = \frac{1}{0.3 - j0.1} = \frac{0.3}{(0.3)^2+(0.1)^2} + j\frac{0.1}{(0.3)^2+(0.1)^2}$$

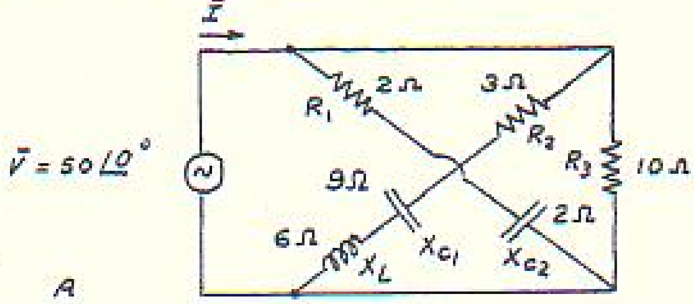
$$\therefore \bar{Z}_T = 3 + j1$$

For check

$$\bar{V} = \bar{I}\bar{Z}_T$$
$$= (60 - j20)(3 + j1)$$
$$= 180 - j60 + j60 + 20$$
$$= 200 + j0 = 200\angle 0^\circ$$

Example (HW)

: Find the current I in the circuit shown in the Fig. -



Answer

$\bar{I} = 33.201\angle 38.89^\circ$ A

Example

: The load taken from a supply consists of :(a). lamp load of 10 kW at unity power factor , (b). motor load of 80 kVA at 0.8 power factor (lag) , and (c). motor load of 40 kVA at 0.7 power factor leading.

Calculate the total load taken from the supply in kW and in kVA and the power factor of the combined load.

Solution

:

ملاحظة : من المناسب حل هذا المثال ان يكون بشكل جدول يضم انواع القدرة المختلفة لمركبات الحمولة وكما يأتي :

Load	kVA	$\cos\phi$	$\sin\phi$	kW	kVAR	
(a)	10	1	0	10	0	(p.f = 1)
(b)	80	0.8	0.6	64	− 48	(p.f lag)
(c)	40	0.7	0.714	28	+ 28.6	(p.f lead)
			TOTAL →	102	− 19.4	

⇒

TS7

$\therefore$ Total kW = 102 $\Rightarrow P_T$

Total kVAR = − 19.4 (lagging) $\Rightarrow Q_T$

$\therefore$ Total kVA taken from the supply S_T

$$S_T = \sqrt{P_T^2 + Q_T^2}$$

$$= \sqrt{(102)^2 + (-19.4)^2} = 103.9 \text{ kVA}$$

and the power factor $= \cos\phi = \dfrac{P_T}{S_T} = \dfrac{102}{103.9} = 0.982$

ويمكن لنا حل المثال أكثر اختصاراً كما يأتي

$$S_T = S_1 + S_2 + S_3$$
$$= P_1 + jQ_1 + P_2 + jQ_2 + P_3 + jQ_3$$
$$= [10\cos 0 + j0 + 80\cos\phi_2 + j\,80\sin\phi_2 + 40\cos\phi_3 + j\,40\sin\phi_3] \times 10^3$$
$$= [(10 + 64 + 28) + j(0 - 48 + 28.6)] \times 10^3$$
$$\therefore \bar{S}_T = 102 - j\,19.4 \quad \text{kVA}$$
$$= 103.9\,\underline{/-10.77} \quad \text{kVA}$$

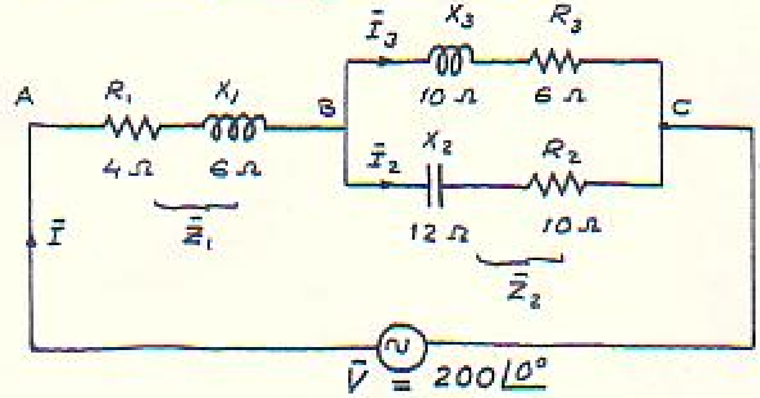
$\therefore$ S = 103.9 kVA

p.f = cos(−10.77)

= 0.982 (lagging)

Example

: Determine the current drawn by the following circuit, where a voltage of 200 V is applied across its terminals. Draw the phasor diagram, and determine the power factor of the circuit.



Solution

: $\bar{Z}_1 = 4 + j6 = 7.2\,\underline{/56.3°}$ Ω

$\bar{Z}_2 = 10 - j12 = 15.6\,\underline{/-50.2°}$ Ω

$Z_3 = 6 + j10 = 11.7\,\underline{/58°}$ Ω

TS7

$\therefore \quad \bar{Z}_T = \bar{Z}_1 + \bar{Z}_{23}$

$$\bar{Z}_{23} = \frac{\bar{Z}_2 \bar{Z}_3}{\bar{Z}_2 + \bar{Z}_3}$$

$$= \frac{(10-j12)(6+j10)}{(16-j2)} = 10.9 + j3.1$$

$$= 11.3 \angle 15.9^\circ \ \Omega$$

$\Rightarrow \bar{Z}_T = (4+j6) + (10.9 + j3.1)$

$= 14.9 + j9.1 = 17.5 \angle 31.4^\circ \ \Omega$

* Now, finding $\bar{I} = ?!$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{200 \angle 0^\circ}{17.5 \angle 31.4^\circ} = 11.4 \angle -31.4^\circ \ A$$

* To draw the phasor diagram, we have (till now) $\bar{V}$ and $\bar{I}$, then we have to find the following quantities :

* $\bar{V}_{AB} = \bar{I}\bar{Z}_1 = (11.4 \angle -31.4^\circ)(7.2 \angle 56.3^\circ)$

$= 82.2 \angle 24.9^\circ$ volts

* $\bar{V}_{BC} = \bar{I}\bar{Z}_{123} = (11.4 \angle -31.4^\circ)(11.3 \angle 15.9^\circ)$

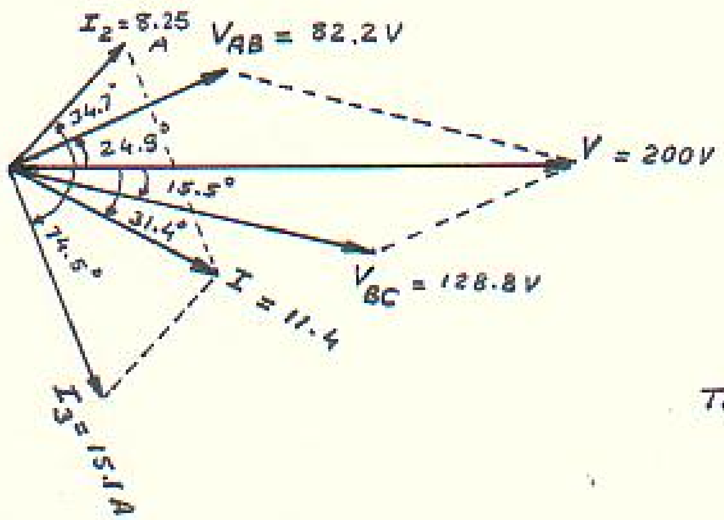
$= 128.8 \angle -15.5^\circ$ volts

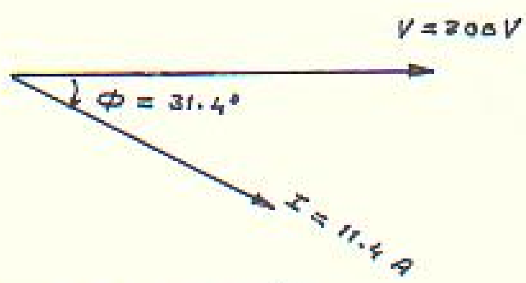
* $\bar{I}_2 = \dfrac{\bar{V}_{BC}}{\bar{Z}_2} = \dfrac{128.8 \angle -15.5^\circ}{15.6 \angle -50.2^\circ}$

$= 8.25 \angle 34.7^\circ$ A

* $\bar{I}_3 = \dfrac{128.8 \angle -15.5^\circ}{11.7 \angle 58^\circ}$

$= 15.1 \angle -74.5^\circ$ A





The power factor is
$\cos\phi = \cos 31.4^\circ$
$= \underline{0.85}$ lagging

8. Resonance in AC Circuits

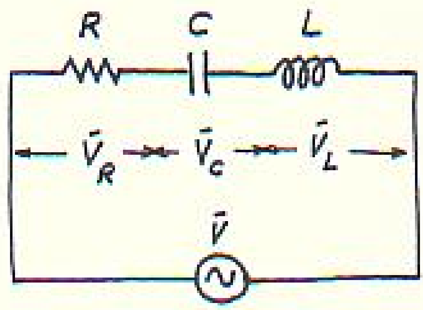
EEB

8.1 Resonance in Series AC Circuits

: Consider the circuit shown;

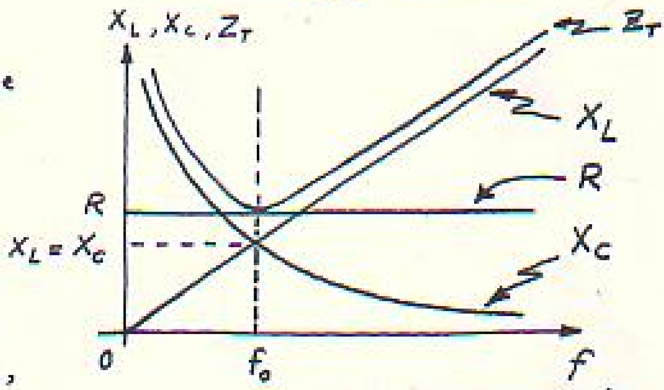
* we have;

$$\bar{Z}_T = R + jX_L - jX_C$$



* At certain frequency (f_o), in the frequency response, we have:

$$\boxed{X_L = X_C}$$



Graphical Representation of Resonance.

* This frequency (f_o) is called the resonance frequency. Then, at this frequency:

$$\boxed{\bar{Z}_T = R}$$ ⇒ and hence;

$$\boxed{\bar{V} = \bar{V}_R = \bar{I}R}$$

* The frequency at which resonance takes place, can be obtained as:

$$X_L = X_C$$

$$\omega_o L = \frac{1}{\omega_o C} \Rightarrow \omega_o^2 = \frac{1}{LC} \Rightarrow \omega_o = \frac{1}{\sqrt{LC}}$$

$$\boxed{\therefore \; f_o = \frac{1}{2\pi\sqrt{LC}}}$$

f_o is in (Hz) if L is in Henry and C in Farad.

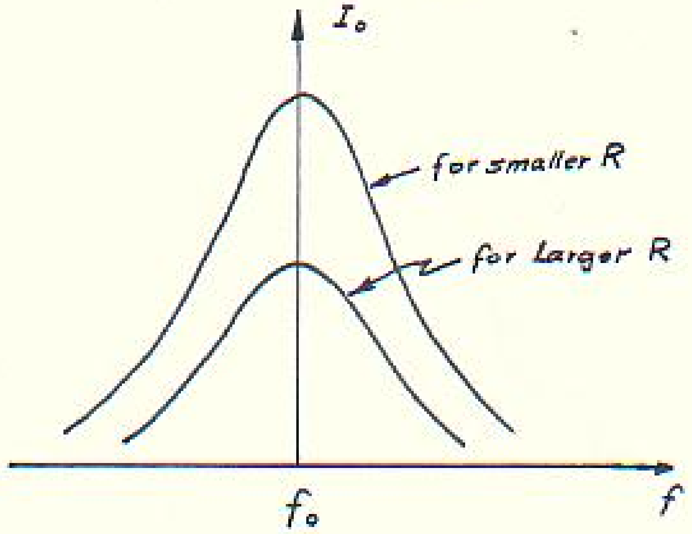
* Some points to remember (when an R-L-C circuit in resonance):

٧ نقاط يجب تذكرها .

- The overall (total) reactance of the circuit is zero (ie, $X_L - X_C = 0$).
- The circuit impedance is minimum (ie, $Z_T = R$).
- Circuit current is maximum, ($I_o = \frac{V}{Z_o} = \frac{V}{R}$).
- Circuit power factor angle is $0°$ $\Rightarrow$ ie, the power factor = 1.
- At resonance $\omega^2 LC = 1$.
- The quality factor Q_o (at resonance) = $\tan\phi = \frac{\omega_o L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$

Current at Resonance $\Rightarrow$

* The Resonance Curve (Frequency Response) :

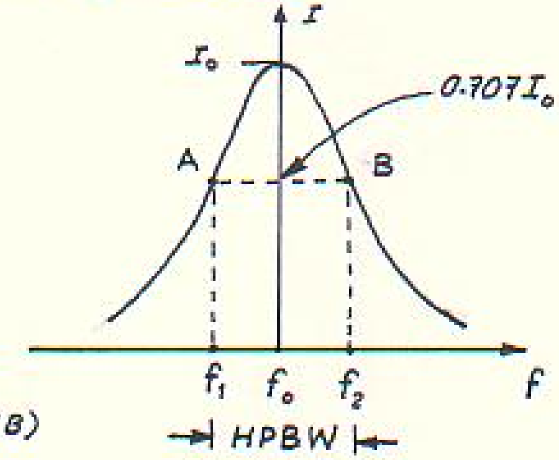


Typical Frequency Response (Resonance Curve) for a series R-L-C circuit in Resonance.

$$\text{Bandwidth} = \Delta\omega = \omega_2 - \omega_1 \Rightarrow \boxed{\Delta f = f_2 - f_1}$$

The narrower the bandwidth, the higher the selectivity of the circuit and vice-versa.

$$P_A = P_B = \left(\frac{I_o}{\sqrt{2}}\right)^2 R = \frac{I_o^2 R}{2} = \frac{P}{2}$$



* Bandwidth is often called (HPBW) to denote the bandwidth at which half the power takes place.

* Also this bandwidth is called (-3dB) bandwidth for the same reason but in a logarithmic scale. (-3dB comes from 10 log 0.5).

In Summary: For an R.L.C circuit in resonance, the following remarks regarding the points A and B in the frequency response:

- Current is $\frac{I_o}{\sqrt{2}} = 0.707\, I_o$
- Impedance is $\sqrt{2}\,R$ or $\sqrt{2}\,Z_o$
- $P_A = P_B = \frac{P_o}{2}$
- The circuit phase angle is $\phi = \pm 45^\circ$
- The quality factor $= Q = \tan\phi = \tan 45^\circ = 1$
- $HPBW = f_2 - f_1$

* How to find f_2 and f_1

* At lower half power frequencies; $\omega_1 < \omega_o \Rightarrow \omega_1 L < \frac{1}{\omega_1 C}$ and $\phi = 45^\circ$.

$$\therefore \frac{1}{\omega_1 C} - \omega_1 L = R \quad \Rightarrow \quad \omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} = 0$$

putting $\frac{\omega_o}{Q_o} = \frac{R}{L}$ and $\omega_o^2 = \frac{1}{\sqrt{LC}}$ in the last equation, then

$$\therefore \omega_1^2 + \frac{\omega_o}{Q_o}\omega_1 - \omega_o^2 = 0$$

The positive solution of the above equation is

$$\omega_1 = \omega_o\left[\sqrt{1 + \frac{1}{4Q_o^2}} - \frac{1}{2Q_o}\right]$$

* Similarly at the upper half power frequencies $\omega_2 > \omega_o$, the positive solution for ω_2 will be as:

$$\omega_2 = \omega_o\left[\sqrt{1 + \frac{1}{4Q_o^2}} + \frac{1}{2Q_o}\right]$$

* for larger values of Q_o (typically greater than 10), the factor $\frac{1}{4Q_o^2}$ becomes negligible compared to 1, then:

$$\omega_1 = \omega_0 \left(1 - \frac{1}{2Q_0}\right)$$
$$= \omega_0 \left(1 - \frac{1}{2\frac{\omega_0 L}{R}}\right)$$
$$= \omega_0 - \frac{\omega_0}{2\frac{\omega_0 L}{R}}$$
$$\therefore \omega_1 = \omega_0 - \frac{R}{2L}$$
$$\Rightarrow \boxed{f_1 = f_0 - \frac{R}{4\pi L}}$$

$$\boxed{\omega_1 = \omega_0\left(1 - \frac{1}{2Q_0}\right)}$$

the lower frequency limit.

and ;

$$\boxed{\omega_2 = \omega_0\left(1 + \frac{1}{2Q_0}\right)}$$

the upper frequency limit.

Similarly

$$\boxed{f_2 = f_0 + \frac{R}{4\pi L}}$$

(*) $$\boxed{\therefore HPBW = f_2 - f_1 = \frac{R}{2\pi L}} \Rightarrow \boxed{HPBW = f_0/Q_0}$$

Q- Factor of a resonant series circuit: There are different ways to derive (or to define) the quality factor Q_0 of a series resonant circuit.

* It is given by the voltage magnification produced in the circuit at resonance ;

we have : $\bar{I}_0 = \frac{\bar{V}}{R}$ at resonance

$$\therefore Q_0 \Rightarrow \text{voltage magnification is} = \frac{V_{L0}}{V}$$

$$\Rightarrow \frac{V_{L0}}{V} = \frac{I_0 X_{L0}}{I_0 R} = \frac{\text{Reactive Power}}{\text{Active Power}} = \frac{X_{L0}}{R} = \underline{\underline{\frac{\omega_0 L}{R}}}$$
$$= \frac{\text{Reactance}}{\text{Resistance}}$$

OR

$$Q_0 = \text{voltage magnification} = \frac{V_{C0}}{V} = \frac{I_0 X_{C0}}{I_0 R} = \frac{X_{C0}}{R}$$
$$= \frac{\text{Reactive Power}}{\text{Active Power}} = \underline{\underline{\frac{1}{\omega_0 C R}}}$$

** The quality factor can also be defined as:

الطاقة المخزونة في الملف $= \frac{1}{2}LI^2$

$$Q_0 = 2\pi \frac{\text{Maximum Energy stored}}{\text{Energy dissipated per cycle}}$$
$$= 2\pi \cdot \frac{\frac{1}{2}LI^2}{I_0^2 R \cdot T_0} = 2\pi \left[\frac{\frac{1}{2}L(\sqrt{2}\,I_0)^2}{I_0^2 R\left(\frac{1}{f_0}\right)}\right]$$

$$\boxed{\therefore Q_0 = \frac{I_0^2 (2\pi f_0) L}{I_0^2 R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}}$$

*** OR, the quality factor (Q_o at resonance), can be Obtained as:

we have: $f_o = \dfrac{1}{2\pi\sqrt{LC}} \Rightarrow \omega_o = \dfrac{1}{\sqrt{LC}}$

and $Q_o = \dfrac{\omega_o L}{R} = \dfrac{\frac{1}{\sqrt{LC}} \cdot L}{R} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

$$\boxed{\therefore\ Q_o = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

NOTE: *Higher values of the Q-factor mean not only higher voltage magnification but also mean high selectivity of the tuning circuit (resonant circuit).*

$\therefore$ In Summary

$$Q_o = \frac{\omega_o L}{R} = \frac{1}{\omega_o C R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \sqrt{\frac{X_{Lo} X_{Co}}{R}}$$

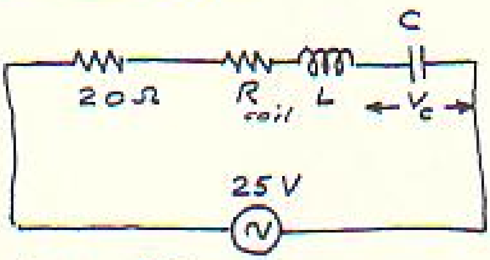
$$= \frac{f_o}{HPBW} = \frac{f_o}{f_2 - f_1}$$

Example: A 20 Ω resistor is connected in series with an inductor, a capacitor and an ammeter across a 25 V supply with variable frequency. When the frequency is 400 Hz, the current is at its maximum value of 0.5 A, and the potential difference across the capacitor is 150 V. Calculate:

(a). the capacitance of the capacitor.
(b). the resistance and the inductance of the inductor.

Solution: $\because$ the current is maximum $\Rightarrow \therefore$ the cct is in resonance

$$\therefore X_c = V_c / I_o = 150/0.5 = 300\ \Omega$$



(a). $X_c = \dfrac{1}{2\pi f_o C} \Rightarrow C = \dfrac{1}{2\pi f X_c}$

$\therefore\ C = \dfrac{1}{2\pi(400)(300)} = 1.3\ \mu F$

(b). $X_L = X_C = 300\ \Omega = 2\pi f_o L \Rightarrow L = \dfrac{300}{2\pi(400)} = 0.119\ H$

* Now R_{coil}? $\Rightarrow$

at resonance Z_T = Resistance of the circuit

$$\therefore \frac{V}{I_o} = R_T$$

$$\therefore \frac{25}{0.5} = 20 + R_{coil}$$

$$\therefore R_{coil} = 50 - 20 = \underline{30\,\Omega}$$

<u>Example</u>: An RLC circuit consists of a series resistance of $1\,k\Omega$, an inductance of 100 mH, and a capacitor of 10 pF. If a voltage of 100 V is applied across the combination, find:

(a). the resonance frequency.
(b). Q-factor of the circuit.
(c). the half-power points.
(d). The half-power bandwidth of the resonance frequency response.

<u>Solution</u>:

(a). The resonance frequency $f_o = ?$

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100\times10^{-3}\times10\times10^{-12}}}$$

$$= \underline{\underline{159\ kHz}}$$

(b). The quality factor of the circuit $Q_o = ?$

$$Q_o = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{1000}\sqrt{\frac{100\times10^{-3}}{10\times10^{-12}}}$$

$$\therefore Q_o = \underline{\underline{100}}$$

(c). The half-power points $\Rightarrow f_1 = ?$ & $f_2 = ?$

$$* \ f_1 = f_o - \frac{R}{4\pi L} = (159\times10^{3}) - \frac{1000}{4\pi\times100\times10^{-3}}$$

$$\therefore f_1 = \underline{\underline{158.2\ kHz}}$$

$$* \ f_2 = f_o + \frac{R}{4\pi L} = (159\times10^{3}) + \frac{1000}{4\pi\times100\times10^{-3}}$$

$$\therefore f_2 = 159.8\ kHz$$

OR

ملحوظة: يمكن حساب HPBW كالآتي :

$$HPBW = f_2 - f_1$$
$$= f_o + \frac{R}{4\pi L} - f_o + \frac{R}{4\pi L}$$

$$\boxed{\therefore \Rightarrow HPBW = \frac{R}{2\pi L}}$$

طرق وتحصل على النتيجة نفسها .

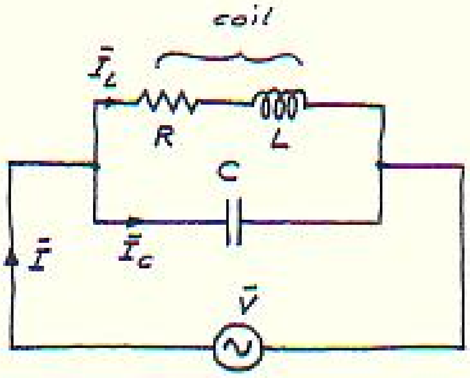
(d). The half power bandwidth HPBW = ?

$$HPBW = f_2 - f_1 = 159.8 - 158.2 = \underline{1.6\ kHz}$$

8.2 Resonance in Parallel Circuits

Consider the parallel RLC circuit shown;

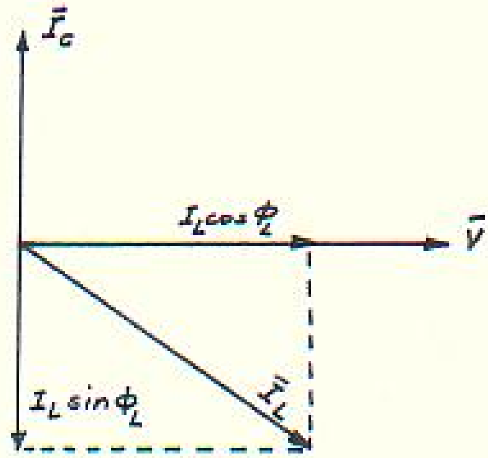
* For this circuit, the resulting phasor diagram can be obtained as shown in the Fig. below



* The CONDITION of resonance for this circuit takes place, when the two reactive components of the line current are EQUAL. This means that:

$$\bar{I}_c = \bar{I}_L \sin\phi_L$$

$$\boxed{\text{or} \quad \bar{I}_c - I_L \sin\phi_L = 0}$$

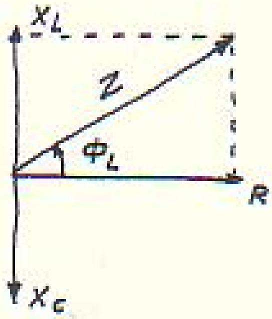


* In terms of impedance, at resonance:

$$Z \sin\phi_L = X_L = X_c$$

$$\Rightarrow X_L = X_c \quad \Rightarrow \quad X_L \cdot X_c = Z^2$$

$$\omega_o L \cdot \frac{1}{\omega_o C} = Z^2$$



$$\left.\begin{array}{l} \dfrac{\omega_o L}{\omega_o C} = Z^2 \\ \text{But } \bar{Z} = R + jX_L \Rightarrow Z^2 = R^2 + X_L^2 \end{array}\right\} \Rightarrow \frac{\omega_o L}{\omega_o C} = R^2 + X_L^2$$

$$\therefore \frac{L}{C} = R^2 + (2\pi f_o)^2 L^2 \quad \Rightarrow \quad (2\pi f_o)^2 L^2 = \frac{L}{C} - R^2$$

$$\boxed{\therefore \quad f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

* يلاحظ انه عندما تكون المقاومة R صغيرة جداً فان : $f_o = \frac{1}{2\pi\sqrt{LC}}$ كما في حالة رنين دوائر التوالي .

* Current at Resonance

Since the net reactive components of the current, at resonance, is zero, then;

$$\bar{I}_o - \bar{I}_L \sin\phi_L = 0$$

Thus the resultant current at resonance is only the real component (see the phasor diagram) which is:

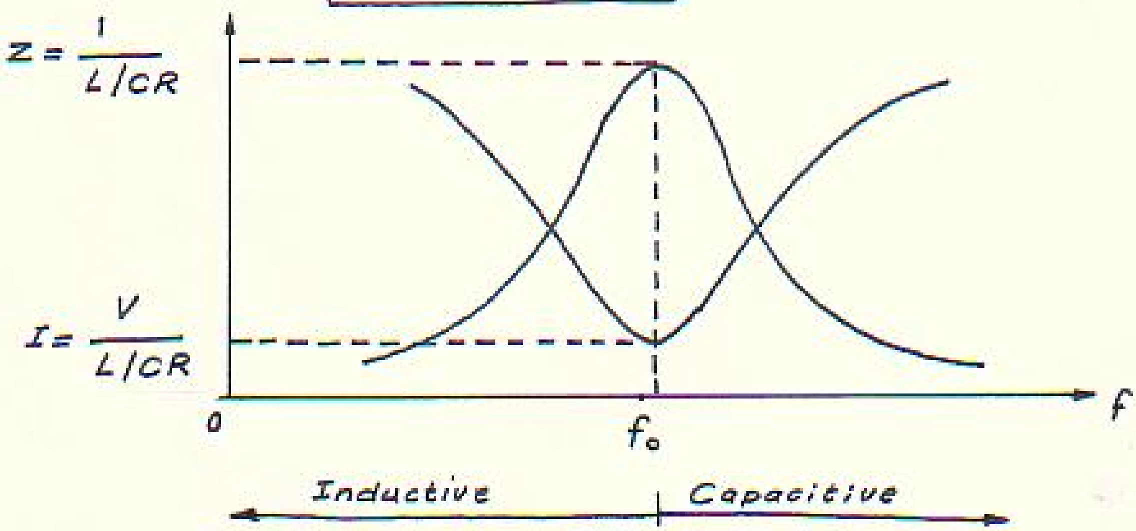
$$\boxed{I_o = I_L \cos\phi_L} \qquad \Rightarrow \quad \bar{I}_L = \frac{\bar{V}}{Z}$$

$$\& \quad \cos\phi_L = \frac{R}{Z}$$

$$\therefore \quad I_o = I_L \cos\phi_L = \frac{VR}{Z^2}$$

$$\text{but} \quad Z^2 = \frac{L}{C} \quad \Rightarrow \quad I_o = \frac{VR}{L/C} = \frac{V}{L/CR}$$

$$\boxed{\therefore \; I_o = \frac{V}{L/CR}}$$ current at resonance

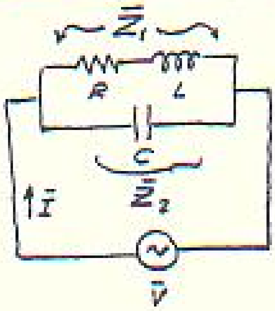


The impedance $\left[Z = \frac{1}{L/CR} \right]$ is called sometimes the effective impedance OR "the dynamic impedance".

طريقة اخرى لحساب f_0 وهي الطريقة العامة

Alternative Method

using the admittance ;



$$\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{R + jX_L} = \frac{R}{R^2+X_L^2} - j\,\frac{X_L}{R^2+X_L^2}$$

$$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{-jX_C} = j\,\frac{1}{X_C}$$

$\bar{Y}_T = G + jB$

$$\therefore \quad \bar{Y}_T = \bar{Y}_1 + \bar{Y}_2$$

$$\therefore \quad \bar{Y}_T = \frac{R}{R^2+X_L^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2}\right)$$

(*) In general, any circuit to be in resonance, the imaginary part (j-component) of the circuit impedance or admittance is zero,

Thus ; for our circuit mentioned earlier ;

$$\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2} = 0$$

$$\Rightarrow \frac{X_L}{R^2+X_L^2} = \frac{1}{X_C} \qquad \Rightarrow X_L \cdot X_C = R^2 + X_L^2 = Z^2$$

وهي نفس النتيجة التي تم الحصول عليها سابقاً

* Talking in terms of suceptance for parallel circuits, the net susceptance is zero at resonance condition. ($B = 0$).

It may be noted that at resonance, the admittance is equal to the conductance (G)

Points to Remember

: The following points about parallel resonance should be noted and compared with those about series resonance. At resonance :

- Net succeptance is zero ($B_T = 0$).
- The admittance equal to the conductance.
- Reactive component of the line current in zero.
- Dynamic impedance = L/CR
- Line current at resonance is minimum and equal $\dfrac{V}{L/CR}$ but it is phase with the applied voltage.
- Power factor of the circuit is unity.

Bandwidth of the Parallel Resonant Circuit

: The bandwidth of a parallel circuit is defined in the same way as that for a series circuit. This circuit also has upper and lower half-power frequencies where power dissipated is half of that at resonant frequency.

Quality Factor of a Parallel Circuit

: It is defined as the current magnification (in the C-branch or in the coil branch) with respect to the line current drawn from the supply. This means that:

$$\text{Q-factor at resonance} \Rightarrow Q_0 = \frac{\bar{I}_c}{\bar{I}}$$

$$\bar{I}_c = \frac{\bar{V}}{X_c} = \frac{\bar{V}}{1/\omega C} = \omega C \bar{V}$$

and

$$\bar{I} = \frac{\bar{V}}{L/CR}$$

$$\therefore Q_0 = \frac{\omega C V}{\frac{V}{L/CR}} = \omega C V \times \frac{L/CR}{V}$$

$$\boxed{\therefore Q_0 = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R}}$$

which is the same as that for series circuits.

$$\therefore Q_0 = \tan\phi$$

Other expressions relating Q_0 can be used in parallel circuits as had been used for series circuits.

Example

: A capacitor is connected in parallel with a coil having $L = 5.52$ mH and $R = 10\,\Omega$, to a 100 V, 50 Hz supply. Calculate the value of the capacitance for which the current taken from the supply is in phase with the applied voltage.

Solution

: Since the current taken from the supply is in phase with the applied voltage $\Rightarrow$ The circuit is in resonance.

Then, at resonance;

$$Z^2 = \frac{L}{C} \quad \text{or} \quad C = L/Z^2$$

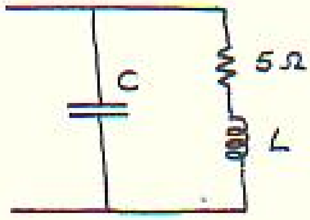
$$X_L = 2\pi f L = 2\pi(50)\, 5.52 \times 10^{-3} = 1.734\ \Omega$$

$$Z = \sqrt{R^2 + X_L^2} \Rightarrow \quad \therefore Z^2 = 10^2 + 1.734^2 \Rightarrow Z = 10.1\ \Omega$$

$$\therefore C = \frac{L}{Z^2} = \frac{5.52 \times 10^{-3}}{(10.1)^2} = 54.6\ \mu F$$

Example

: Calculate the impedance of the parallel-tuned circuit shown at a frequency of 500 kHz and fo a bandwidth of operation equal to 20 kHz. The resistance of the coil is 5 Ω.



Solution

: $$HPBW = \frac{R}{2\pi L} = 20\times10^{3} \ Hz$$

$$\therefore L = \frac{5}{2\pi(20\times10^{3})} = 39\ \mu H$$

$$f_o = 500\times10^{3}\ Hz = \frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}} = \frac{1}{2\pi}\sqrt{\frac{1}{39\times10^{-6}C}-\frac{5^2}{(39\times10^{-6})^2}}$$

$$\therefore C = 2.6\times10^{-9}\ F = 2.6\ nF$$

$$Z = \frac{L}{CR} = \frac{39\times10^{-6}}{2.6\times10^{-9}\times5} = 3\ k\Omega.$$

ملاحظة : في حال اضافة اي عنصر الى الدائرة المذكورة في المثال ، فلا بد من
استخدام العلاقة الخاصة بـ (f_o) كما تعلمنا سابقاً .

Tutorial Sheet № 8

TS8

Example: For a series R.L.C circuit, the inductor is variable. The source voltage is $(\sqrt{2}\, 200 \sin 100\pi t)$. Maximum current obtained by varying the inductance is 0.314 A and the voltage across the capacitor is 300 V. Find the circuit elements.

Solution: Max. current $\Rightarrow$ Resonant conditions, then:

$$I_m = I_o = \frac{V}{R} \qquad \text{and also} \qquad V_L = V_C$$

$$\therefore \quad R = \frac{V}{I_m} = \frac{200}{0.314} = \underline{637\ \Omega}$$

$$V_C = I_o X_C = \frac{I_o}{\omega_o C} \qquad\qquad \omega_o = 100\pi$$

$$\therefore \quad C = \frac{I_o}{\omega_o V_C} = \frac{0.314}{(100\pi)(300)} = \underline{3.33\ \mu F}$$

$$V_L = I_o X_L = I_o (2\pi f_o) L = I_o \omega_o L$$

لوحظ ان
$V_L = V_C$
at Resonance

$$\therefore \quad L = \frac{V_L}{\omega_o I_o} = \frac{300}{(100\pi)(0.314)} = \underline{3.03\ H}$$

Example: A coil having an inductance of 50 mH and a resistance of 10 Ω is connected in series with a 25 μF capacitor across a 200 V ac supply. Calculate:

(a). resonance frequency of the circuit.
(b). current flowing at resonance.
(c). the value of Q_o using different expressions.

Solution: (a).

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{50\times10^{-3} \times 25 \times 10^{-6}}}$$

$$= 142.3\ Hz.$$

TSB

(b). $I_o = \dfrac{V}{R} = \dfrac{200}{10} = 20 \text{ A}$

ملاحظة: في هذا المثال ان Q_o (معامل جودة دائرة الرنين) هو نفسه Q (معامل جودة الملف) لانه لا توجد مقاومات اضافية اخرى عدا مقاومة الملف؛ لاحظ ذلك مستقبلاً حيث ان اية مقاومة اضافية (حسب موقعها في الدائرة) تؤثر في قيمة Q_o.

(c). $Q_o = \dfrac{\omega_o L}{R} = \dfrac{2\pi \times 142.3 \times 50\times10^{-3}}{10} = 4.47$

<u>or</u>

$$Q_o = \frac{1}{\omega_o C R} = \frac{1}{2\pi \times 142.3 \times 25\times10^{-6} \times 10} = 4.47$$

or

$$Q_o = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{50\times10^{-3}}{25\times10^{-6}}} = 4.47$$

<u>Example</u>:

A series R-L-C circuit consists of R = 1000 Ω, L = 100 mH and C = 10 pF. The applied voltage across the circuit is 100 V. Then;

(a). Find the resonant frequency of the circuit.
(b). Find the quality factor of the circuit at the resonant freq.
(c). At what angular frequencies do the half power points occur?
(d). Calculate the HPBW of the resonance curve.

<u>Solution</u>:

ملاحظة: الاصل في الحل ان تشتق (f_o)!؟

(a). $f_o = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{100\times10^{-3} \times 10\times10^{-12}}} = \underline{159.15 \text{ kHz}}$

(b). $Q_o = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{1000} \times \sqrt{\dfrac{1000\times10^{-3}}{10\times10^{-12}}} = \underline{100}$

(c). $HPBW = \dfrac{R}{2\pi L} = \dfrac{1000}{2\pi \times 100 \times 10^{-3}} = 1591.5 \text{ Hz}$

او بعلاقة اخرى اكثر عمومية تستخدم لجميع انواع دوائر الرنين.

$\xrightarrow[\text{relation}]{\text{more general}}$ $HPBW = f_o / Q_o = \dfrac{159.15\times10^{3}}{100} = 1591.5 \text{ Hz}$

(d). $\omega_1 = \omega_o\left(1 - \dfrac{1}{2Q_o}\right) = 2\pi \times 159.15\times10^{3}\left(1 - \dfrac{1}{2\times100}\right)$

$= 994.464 \times10^{3}$ rad/sec.

& $\omega_2 = \omega_o\left(1 + \dfrac{1}{2Q_o}\right) = 2\pi \times 159.15\times10^{3}\left(1 + \dfrac{1}{2\times100}\right)$

$= 1004.459 \times10^{3}$ rad/sec.

✱ ملاحظة: من الممكن استخدام العلاقة الاكثر عمومية:

$$\omega_1 = \omega_o - \frac{HPBW}{2}$$

&

$$\omega_2 = \omega_o + \frac{HPBW}{2}$$